The Flexible Phase Entropy and its Rise from the Universal Cybernetics Duality

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Abstract — **In this paper the entropy of a flexible phase (FP) medium (referred as FP entropy) emerges from a novel linger thermo theory (LTT) and then used in a biological lifespan study. LTT is the 'dynamic metrics' dual of the 'stationary metrics' latency information theory (LIT). These two nascent theories are synergistic time/space designs of the universal cybernetics duality (UC duality), first identified in linear quadratic Gaussian (LQG) control in 1978. While LIT has already yielded outstanding solutions for high-performance radar, LTT has done the same for biological lifespan, with both holding US patents. The FP entropy equation is found here subject to a constant internal mass-energy constraint of LTT that reflects actual gravitational/non-gravitational interactions of atoms or molecules. Moreover, this approach is revealed to contain a degrees of freedom (DoF) coupling constant that when multiplied by the heat capacity of liquid water at 310 K in thermal equilibrium accurately models the heat capacity of the human medium in its non-equilibrium thermal state. The use of the DoF coupling constant mechanism has been found to result in outstanding theoretical adult lifespan predictions that are directly linked to an individual's heat capacity. These sensible results compel the view that the FP entropy approach will find broad use in future biophysical chemistry of lifespan studies.**

Keywords — thermodynamics entropy, lingerdynamics ectropy, information entropy, latency ectropy, statistical physics, biophysical chemistry, lifespan, cosmology, certainty, uncertainty, unification

I. Introduction

Life science studies often require the availability of a reliable as well as simple mathematical model for the entropy of biological mediums for use in accurate assessments. One area of research where such a model is highly desirable is in human lifespan studies where lifespan predictions from such a model can be used by medical professionals and the insurance industry. An entropy model that because of its simplicity has been utilized in the past to make such predictions [1] is that for an ideal gas [2]. Unfortunately, however, the predictions derived with such a model are often unreliable because the internal energy associated with the classical ideal gas entropy (IG entropy) expression only reflects the kinetic energy and potential energy of the atoms or molecules in a gas medium. Thus, if such a model is used to describe the entropy of a biological medium such as our own where more than 98% of our molecules are of liquid water at an approximate temperature of 310 *K*, it would fail to model the gravitational/non-gravitational interactions between the molecules of water in its liquid phase. As a result there is a need for a simple entropy model, such as the IG entropy one, which reflects gravitational/non-gravitational interactions between atoms or molecules and thus may be used to better describe the entropy of biological mediums. This is the problem addressed here.

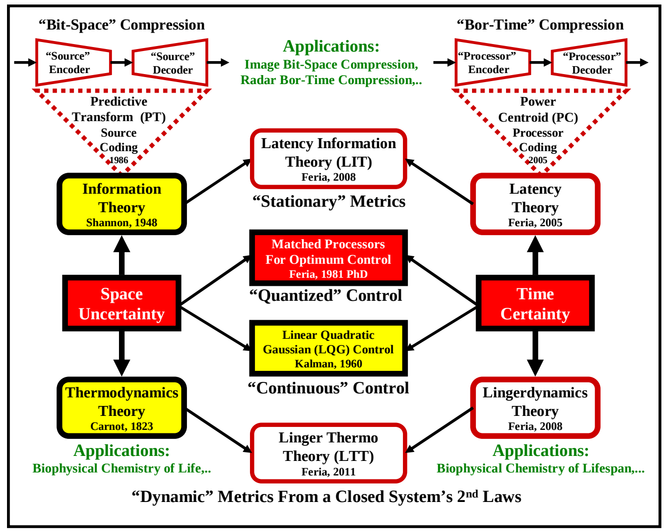

Figure 1. Summary of universal cybernetics duality development

In this paper the entropy of mediums with an arbitrary phase will be modeled by a novel flexible phase entropy (FP entropy) that is both surprisingly simple in its form (it is similar to that of the IG entropy), and is remarkably accurate in its theoretical predictions of biological lifespan. This entropy approach will be shown to inherently emerge from a novel linger thermo theory (LTT) [3], one of two synergistic time/space designs of the universal cybernetics [4] duality (UC duality), see Fig. 1. This duality theory that enables efficient system designs is now briefly surveyed.

The UC duality was first identified by the author in 1978 in linear quadratic Gaussian (LQG) control [5] stating that,

"*Synergistic physical and mathematical dualities naturally arise in efficient system designs*"

More specifically, the "physical duality" conveyed the separation of the system design into a space-uncertainty communication problem (Kalman filter design in LQG) and a time-certainty control problem (linear quadratic controller design in LQG), while the "mathematical duality" conveyed the appearance of identical mathematical structures (Ricatti design equations in LQG) in the separately designed communication and control subsystems. The discovery of the UC duality first led to "Matched Processors (MPs) for Optimum Control", the author's 1981 Ph.D. [6], see Fig. 1. While LQG dealt with continuous control, MPs dealt with quantized control. In MPs the certainty-based parallel structures of the MPs controller were the control's certainty-based time dual of communication's uncertainty-based parallel structures of Matched Filters for bit detection [7]. A remarkable result of Matched Processors [8] was that unlike Bellman's Dynamic Programming [9], it did not suffer of what Bellman called "the curse of dimensionality" when referring to the exponential increase in computational burden as the process state dimension increased in value. After the application of the UC duality to quantized control it was not until much later in the mid 2000s when Defense Advanced Research Projects Agency (DARPA) funded research on high-performance knowledge-aided (KA) adaptive radar [10] led the author to discover the time-certainty dual of space-uncertainty information theory [11], that he named latency theory [12]. Later the time-certainty dual of space-uncertainty thermodynamics was also revealed and named lingerdynamics [1], [13]. These theories were then merged to yield the time/space synergistic latency information theory (LIT) and LTT where each addressed four different types of system functions. These four functions were: 1) a "*source*" uncertainty function measured by a source entropy space-metric (this metric was the Shannon's "info-source" entropy in LIT and the Boltzmann's "thermo-source" entropy in LTT [14]); 2) a "*processor*" certainty function measured by a novel processor ectropy time-metric (ectropy is the time dual of entropy); 3) a "*retainer*" uncertainty function measured by a novel retainer entropy space-metric; and 4) a "*mover*" certainty function measured by a novel mover ectropy time-metric. While the units of the source and processor metrics were mathematical, i.e., in *b*inary dig*it* (*bit*) and *b*inary operat*or* (*bor*) units, respectively, the units of the retainer and mover metrics were physical, e.g., in square meters (m^2) and seconds (*sec*), respectively. Yet the nature of the LIT and LTT space/time metrics is quite different. In the case of LIT they are time invariant, or stationary in nature, while in the LTT case they are time varying, or dynamic in nature. The LTT dynamic property has roots in one of the four laws of thermodynamics [14], or more specifically, the 2nd law of thermodynamics implying that the Boltzmann entropy (or equivalently the thermo-source entropy space-metric) increases with time for a closed system. It can be further shown that similar increases occur to the remaining LTT space/time metrics with the passing of time [13]. In the case of LIT it was first applied to radar design [10], [15], [16] where it yielded a fast and powerful approach to radar that can be either KA or knowledge-unaided (KU) and emulated the signal to interference plus noise ratio (SINR) performance derived with high-performance KA adaptive radar, a surprising result [17].

On the other hand, for the LTT case a universal linger thermo equation (ULTE) was derived [3], given in Section III, that combines "physical" operating ratios (such as the mass ratio $M/\Delta M$ where M is the mass of the medium and ΔM is a quantum of operation (QoO) of the mass, and the lifespan ratio $\tau/\Delta\tau$ where τ is the "lifespan of thermo bits of interest or life-bits" in the medium and $\Delta\tau$ is a QoO lifespan) to yield a lifespan theory for biological systems that is based on the "*quadratic lifespan equation*" $\tau/\Delta\tau = (M/\Delta M)^2$. For example, this theory predicts for an individual weighting M=70 *kg* and with a nutritional consumption rate (NCR) of 1,841 *kcal* per day (or ΔM=0.3628 *kg* of food per day) with $\Delta\tau$ =1 day=1/365 years a *theoretical adult lifespan* (τ) of 102 years where NCR is noted to be a *macro* physical metric. Moreover, when 18 years of childhood are added to τ it yields 120 years of lifespan. Also, if the mass M of the individual is kept constant, the quadratic lifespan equation has the NCR controlling the individual's lifespan. For example if the individual's NCR is increased from 1,841 to 2,827 *kcal* per day his/her theoretical adult lifespan would be decreased to 42 years for a total of 60 years when 18 years of childhood are added. This is a reasonable result since a higher NCR should lead to an increased "metabolic strain" expected to result in the individual aging at a faster rate [18].

In the ULTE the relationship between the aforementioned "physical" operating ratios and "mathematical" quantities such as the thermo-source entropy and the linger-processor ectropy metrics are not explicitly stated since they depend on the entropy expression for the medium. In the case of black holes and photon gases these entropies have been previously derived [3], but not for flexible phase mediums which is done here. The flexible phase entropy or FP entropy will be readily derived by altering the ideal gas entropy expression [2] such that it satisfies an *internal mass-energy* LTT regulation constraint stated in Section II.

The FP entropy approach is advanced in Section III and then its rationality established by showing that it leads to sensible theoretical adult lifespan or τ predictions. These predictions are based on a degrees of freedom (DoF), a *micro* mathematical metric, coupling constant (η) [19], conveying the non-equilibrium thermodynamics state of the medium. For instance, when η is 0.66 a τ of 102 years is predicted (with τ increasing as η decreases in value) for a

70 *kg* individual whose medium is modeled as liquid water at a 310 *K* temperature and DoF equal to 6.

The lifespan prediction approach of the LTT FP entropy method is expected to find broad use in future biophysical chemistry of lifespan studies/applications. This is the case since as mentioned earlier gravitational/non-gravitational interactions of atoms or molecules are the rule rather than the exception for just about all kinds of mediums.

The paper organization is as follows. In Section II LTT is reviewed with its metrics defined. In Section III the LTT metrics are combined to yield the ULTE which is then illustrated for three mediums. In Section IV the FP entropy and IG entropy expressions are contrasted. In Section V the flexible phase ULTE is applied to a human lifespan study. A summary and conclusions section ends the paper.

II. Linger Thermo Theory

In this section LTT is reviewed with the two entropy and two ectropy metrics of its four system functions defined.

LTT studies mediums whose mass-energy:

$$E=Mc^2 \tag{1}$$

is regulated, i.e., it is kept constant, while interacting with its surroundings via black body radiation [3]. *M* denotes the medium mass, *c* the speed of light in a vacuum and *E* the the total medium energy. In addition, LTT assumes that the medium volume has a minimum expected surface area of interaction with its surroundings which in turn defines a sphere of radius (*r*). As a result LTT is characterized by the most efficient type of mass-energy interface between the medium and its surroundings. Finally, the total mass-energy of the medium is assumed to act as a point mass located at the center of the LTT's spherical volume. In this way the perpetual rotational speed (v) of a particle on the sphere's surface as well as its escape speed (v_e) are described according to:

$$v^2 = GM/r \tag{2}$$

$$v_e^2 = 2GM/r \tag{3}$$

and

$$v_e = \sqrt{2}v \tag{4}$$

where *G* is the gravitational constant and (4) is the v to v_e coupling equation.

In LTT four different kinds of system functions characterize all mediums. Two are "thermal-uncertainty space" types and the remaining two are "linger-certainty time" types. While the two thermal functions pertain to the "sourcing and retention" of mass-energy that are measured with two entropy metrics, the two linger functions pertain to the "processing and motion" of mass-energy that are measured with two ectropy metrics.

The two thermo entropies and the two linger ectropies are defined as follows:

A. The Boltzmann thermo-source entropy

The Boltzmann thermo-source entropy ($\hat{H}$) denotes the "amount of thermal-uncertainty bits" of the system microstates according to the following "expectation uncertainty metric" (in *mathematical* bit units):

$$\hat{H} = \sum_{i=1}^{\Lambda_{\hat{H}}} \log_2(1/P[\mu_i])P[\mu_i] = \log_2 \Omega = S/k\ln 2 \tag{5}$$

where $\Lambda_{\hat{H}}$ is the number of *realizations* of a microstate μ_i (describing a microscopic configuration of a thermodynamics system occupied with probability $P[\mu_i]$ in the course of thermal fluctuations). The expression $\log_2(1/P[\mu_i])$ denotes the "amount of thermal-uncertainty bits" associated with μ_i. In addition, $\log_2(1/P[\mu_i])$ denotes the smallest possible number of thermal-uncertainty bits for μ_i. Moreover, Ω in $\hat{H} = \log_2 \Omega$ denotes the 'effective' number of equally likely microstate realizations resulting in $\hat{H}$. When the microstates are equally likely it follows that Ω and $\Lambda_{\hat{H}}$ would be the same. Finally, $\hat{H} = \log_2 \Omega$ $= S/k\ln 2$ linearly relates $\hat{H}$ to the Boltzmann "statistical" thermodynamics entropy (*S*) and constant (*k*), both in *Joules/K* units [3].

B. The thermo-retainer entropy

The thermo retainer entropy ($\hat{N}$) denotes the "amount of thermal-uncertainty square meters" of the system microstates according to the following "expectation uncertainty metric" (in *physical* SI m^2 units):

$$\hat{N} = \sum_{i=1}^{\Lambda_{\hat{H}}} 4\pi \bar{r}_i^2 P[\mu_i] = 4\pi r^2 \tag{6}$$

where $\Lambda_{\hat{H}}$ is the number of *realizations* of a microstate μ_i and $\bar{r}_i$ is the radius of the sphere whose shape the μ_i volume is expected to assume. The expression $4\pi \bar{r}_i^2$ denotes the "amount of thermal-uncertainty square meters" corresponding to the surface area of the μ_i sphere. In addition, $4\pi \bar{r}_i^2$ denotes the smallest possible thermal-uncertainty surface area that an arbitrarily shaped volume for μ_i could have, i.e., that of a sphere. Finally, r in $4\pi r^2$ denotes an average radius for all the microstate spheres.

C. The linger-processor ectropy

The linger-processor ectropy ($\hat{K}$) denotes the "amount of linger-certainty bors" of the system microstates according to the following "minimax certainty metric" (in *mathematical* bor units):

$$\hat{K} = \max\{\log_{C[\mu_1]} h_1, .., \log_{C[\mu_{\Lambda_{\hat{H}}}]} h_{\Lambda_{\hat{H}}}\} = \sqrt{h} \tag{7}$$

where $\Lambda_{\hat{H}}$ is the number of *realizations* of a microstate μ_i, h_i is the number of bits for processing under μ_i and $C[\mu_i]$ is a "constraint" on the maximum number of inputs that a basic mathematical operator (or physical gate) can have under μ_i. The expression $\log_{C[\mu_i]} h_i$ denotes the "amount of linger-certainty bors" associated with μ_i where the approximation $\log_{C[\mu_i]} h_i \cong \sqrt{h_i}$ holds when $C[\mu_i]$ approaches the value of one and h_i is a very large number. In addition, $\log_{C[\mu_i]} h_i$ denotes the smallest possible amount of linger-certainty bors of processing under μ_i, see [13] for the derivation of the $\log_{C[\mu_i]} h_i$ expression and its illustration using a simple full adder example [21]. Finally under the condition $\log_{C[\mu_i]} h_i \cong \sqrt{h_i}$ for all i the h in $\hat{K} = \sqrt{h}$ denotes the maximum number of thermo-bit inputs linked to the microstate realization whose number of linger bors is the same as $\hat{K}$.

D. The linger-mover ectropy

The linger-mover ectropy ($\hat{A}$) denotes the "amount of linger-certainty seconds" of the system microstates according to the following "minimax certainty metric" (in *physical* SI sec units):

$$\hat{A} = \max\{\pi\bar{r}_1/v_1,..,\pi\bar{r}_{\Lambda_{\hat{H}}}/v_{\Lambda_{\hat{H}}}\} = \pi r/v \qquad (8)$$

where $\Lambda_{\hat{H}}$ is the number of *realizations* of a microstate μ_i and $\bar{r}_i$ is the expected radius of the sphere where μ_i resides when the expected shape of its volume is that of a sphere. The expression $\pi\bar{r}_i/v_i$ denotes the "amount of linger-certainty seconds" corresponding to one half of a circular rotational motion on the surface of a sphere of radius $\bar{r}_i$ with v_i denoting the rotational speed of motion in μ_i. In addition, $\pi\bar{r}_i/v_i$ denotes the smallest possible linger-certainty seconds for rotational motion since v_i is the largest possible in value [13]. Finally r and v in $\hat{A}=\pi r/v$ denote the average radius and average rotational speed for all microstate spheres, respectively.

III. UNIVERSAL LINGER THERMO EQUATION

In this section the LTT metrics (5)-(8) are combined to yield the universal linger thermo equation or ULTE which is then illustrated with black hole, photon gas and flexible phase mediums. The entropies (5) and (6) and the ectropies (7) and (8) when combined produce the ULTE which is a "general medium operational expression" according to [3]:

$$\hat{H} = \log_2 \Omega = g_{Med}\left(\frac{\hat{N}}{\Delta\hat{N}} = \frac{V}{\Delta V} = \frac{\tau}{\Delta\tau} = \left(\frac{r}{\Delta r}\right)^2 = \left(\frac{M}{\Delta M}\right)^2 = \left(\frac{\hat{A}}{\Delta\hat{A}}\right)^2\right) = \hat{K}^2 \qquad (9)$$

where g_{Med} is a function of the medium under study and relates the source/processor metrics pair $(\hat{H},\hat{K})$ (characterized by mathematical units) to dimensionless operating ratios of physical variables, inclusive of the retainer/mover metrics pair $(\hat{N},\hat{A})$. An example of a dimensionless operating ratio is $M/\Delta M$ with $M=E/c^2$ denoting the mass-energy of the medium (whose value is regulated to remain constant) and ΔM is a QoO mass representing an active or operating portion of M. The following three relationships are next highlighted regarding the ULTE:

1) The "mathematical units" entropy/ectropy equation:

$$\hat{H} = \hat{K}^2 \qquad (10)$$

that surfaces from (7) when h is replaced with $\hat{H}$.

2) The "physical units" entropy/ectropy relationship (augmented with additional physical variables):

$$\hat{N} = 4\pi r^2 = 3\frac{4\pi r^3/3}{r} = 3\frac{V}{r} = 4\pi\left(\frac{GM}{v^2}\right)^2 = \frac{3\tau}{r\Pi} = \frac{4v^2\hat{A}^2}{\pi} \qquad (11)$$

that surfaces from the use of: a) the equations for the thermo-retainer entropy (6) and the linger-mover ectropy (8); b) the equation for the perpetual rotational speed (2); and c) the equation for the "life-bits pace (Π)" defined according to (in SI *sec*/m^3 units):

$$\Pi = \tau/V = 3\tau/r\hat{N} \qquad (12)$$

where τ is the retention time of "thermo-bits of interest or life-bits" that defines a portion of the medium that leaves its expected spherical volume (V) via black-body radiation under the assumption that it never returns to once again perform its original "life-bit" function (with the caveat that its mass-energy is fully recovered from the medium's surroundings since $E=M/c^2$ is regulated in LTT to have a constant value). An example of "life-bits for a non-living system" are the thermo-bits of some compressed synthetic aperture radar (SAR) image residing in a medium that also contains the thermo-bits of the source-coder that derived the image [10]. Another example is of "life-bits for a living system" responsible for the day to day survival of an organism in a medium that also contains the thermo-bits that give the organism structure.

3) The following "physical units" QoO composite expression:

$$\Delta\hat{N} = 4\pi\Delta r^2 = 3\frac{4\pi r\Delta r^2/3}{r} = 3\frac{\Delta V}{r} = 4\pi\left(\frac{G\Delta M}{v^2}\right)^2 = \frac{3\Delta\tau}{r\Pi} = \frac{4v^2\Delta\hat{A}^2}{\pi} \qquad (13)$$

that is the QoO version of (11).

The ULTE is now stated for black hole, photon gas and flexible phase mediums.

A. The Black Hole ULTE

The black hole (BH) ULTE is given according to [3]:

$$\hat{H}_{BH} = \frac{\hat{N}_{BH}}{\Delta\hat{N}_{BH}} = \frac{V_{BH}}{\Delta V_{BH}} = \frac{\tau_{BH}}{\Delta\tau_{BH}} = \left(\frac{r_{BH}}{\Delta r_{BH}}\right)^2 = \left(\frac{M_{BH}}{\Delta M_{BH}}\right)^2 = \left(\frac{\hat{A}_{BH}}{\Delta\hat{A}_{BH}}\right)^2 = \hat{K}^2_{BH} \qquad (14)$$

$$\Delta\hat{N}_{BH} = \ln 2 \frac{1920}{\chi c} = 7.2534 \text{ x } 10^{-70} \ m^2 \quad (15)$$

$$S_{BH} = k\frac{4\pi G}{c^5\hbar}E_{BH}^2 = k\frac{\chi c}{1920}\hat{N}_{BH} = k\ln 2\hat{H}_{BH} = k\ln 2\frac{\hat{N}_{BH}}{\Delta\hat{N}_{BH}} = \ldots \quad (16)$$

$$kT_{BH} = \left(\frac{\partial(S_{BH}/k)}{\partial E_{BH}}\right)^{-1} = k\frac{E_{BH}}{2S_{BH}} = \frac{c^5\hbar}{8\pi G E_{BH}} \quad (17)$$

$$\chi = \frac{\tau_{BH}}{V_{BH}} = 480\frac{c^2}{\hbar G} = 6.1203 \text{ x } 10^{63} \ s/m^3 \quad (18)$$

$$\Delta\tau_{BH} = 640\ln 2 r_{BH}/c \quad (19)$$

$$\Delta M_{BH} = \ln 2 c\hbar/4\pi G = 5.1152 \ \text{ x } 10^{-9} kg \quad (20)$$

$$\Diamond E_{\Delta\tau_{BH}}^{LB=1} = \Diamond M_{\Delta\tau_{BH}}^{LB=1}c^2 = \left(1-\sqrt[3]{1-\Delta M_{BH}^2/M_{BH}^2}\right)M_{BH}c^2 \quad (21)$$

where all the variables in (14)-(21) were either implicitly or explicitly defined earlier in (1)-(13) except for: a) T_{BH} denoting the temperature of the black hole; b) $\hbar$ denoting the reduced Planck constant; c) χ denoting "*pace of dark in a black hole*" [3] (χ is the retention dual of motion's "*speed of light in a vacuum*", noted from (18) to be the ratio of the duration of life-bits in the black hole (τ_{BH}) over its initial volume V_{BH}—with all the thermo-bits in this volume assumed to be life-bits, i.e., thermo-bits of interest); and d) $\Diamond E_{\Delta\tau_{BH}}^{LB=1}$ denotes the quantum of radiation (QoR) energy of the "single" life-bit emitted during the black hole QoO lifespan $\Delta\tau_{BH}$ (19), see [13] for the derivation of (21).

B. The Photon Gas ULTE

The photon gas (PG) ULTE is defined according to [3]:

$$\hat{H}_{PG} = \frac{\hat{N}_{PG}}{\Delta\hat{N}_{PG}} = \frac{V_{PG}}{\Delta V_{PG}} = \frac{\tau_{PG}}{\Delta\tau_{PG}} = \left(\frac{r_{PG}}{\Delta r_{PG}}\right)^2 = \left(\frac{M_{PG}}{\Delta M_{PG}}\right)^2 = \left(\frac{\hat{A}_{PG}}{\Delta\hat{A}_{PG}}\right)^2 = \hat{K}_{PG}^2 \quad (22)$$

$$\Delta\hat{N}_{PG} = \ln 2\frac{135c^3\hbar^3}{4\pi^2(kT)^3 r_{PG}} \quad (23)$$

$$S_{PG} = k\frac{16\pi^3(kT_{PG})^3G^3E_{PG}^3}{135c^9\hbar^3v_{PG}^6} = k\frac{4\pi^2(kT_{PG})^3r_{PG}}{135c^3\hbar^3}\hat{N}_{PG}$$

$$= k\ln 2\,\hat{H}_{PG} = k\ln 2\frac{\hat{N}_{PG}}{\Delta\hat{N}_{PG}} = \ldots \quad (24)$$

$$kT_{PG} = \left(\frac{\partial(S_{PG}/k)}{\partial E_{PG}}\right)^{-1} = k\frac{E_{PG}}{3S_{PG}} = \frac{135c^9\hbar^3v_{PG}^6}{48\pi^3(kT_{PG})^3G^3E_{PG}^2} \quad (25)$$

where all the variables in (22)-(25) were earlier defined, and when applicable are appropriately redefined in the context of a photon gas medium [3].

C. The Flexible Phase ULTE

The flexible phase (FP) ULTE is defined according to:

$$\hat{H} = J\log_2\left(\frac{\hat{N}}{\Delta\hat{N}} = \frac{V}{\Delta V} = \frac{\tau}{\Delta\tau} = \left(\frac{r}{\Delta r}\right)^2 = \left(\frac{M}{\Delta M}\right)^2 = \left(\frac{\hat{A}}{\Delta\hat{A}}\right)^2\right) = \hat{K}^2 \quad (26)$$

$$\Delta\hat{N} = \frac{J^{\alpha\eta}}{e^{5/2}}\left(q^Eq^T\right)^{-1}\left(q^Rq^v\right)^{-\frac{\eta(c_V-3/2)}{2}}\left(\frac{c_V(\eta)JkT}{E}\right)^{c_V(\eta)}\hat{N} \quad (27)$$

$$S/k = \ln\Omega = J\ln\left(\frac{e^{5/2}}{J^{\alpha\eta}}q^Eq^T\left(q^Rq^v\right)^{\frac{\eta(c_V-3/2)}{2}}\left(\frac{E}{c_V(\eta)JkT}\right)^{c_V(\eta)} = \frac{\hat{N}}{\Delta\hat{N}} = \ldots\right) = \ln 2 \ \hat{H} \quad (28)$$

$$q^E = \sum_{\substack{Electronic\\States}} e^{-\frac{\varepsilon_i^E}{kT}} = \sum_{\substack{Electronic\\Energies}} g_i e^{-\frac{\varepsilon_i^E}{kT}} \approx g_0 e^{-\frac{\varepsilon_0^E}{kT}} = ge^{-\frac{5}{2}} \quad (29)$$

$$q^T = \sum_{\substack{Translational\\States}} e^{-\frac{\varepsilon_i^T}{kT}} \approx \frac{V}{\Lambda^3} = V\left(\frac{mkT}{2\pi\hbar^2}\right)^{3/2} \quad (30)$$

$$q^R = \sum_{\substack{Rotational\\States}} e^{-\frac{\varepsilon_i^R}{kT}} \approx \frac{2IkT}{\sigma\hbar^2} \quad (31)$$

$$q^v = \sum_{\substack{Vibrational\\States}} e^{-\frac{\varepsilon_i^v}{kT}} \approx \frac{kT}{hv} \quad (32)$$

$$kT = \left(\frac{\partial(S/k)}{\partial E}\right)^{-1} = \frac{E}{c_V(\eta)J} \quad (33)$$

$$c_V(\eta) = \frac{1}{J}\frac{\partial E}{\partial(kT)} = \frac{kT}{J}\frac{\partial(S/k)}{\partial(kT)} = 3/2 + \eta(c_V - 3/2) \quad (34)$$

$$S = J\frac{\Delta S}{\Delta J} = J\frac{Q/T}{\Delta M/\Delta m} = kJ\frac{Q}{(\Delta M/\Delta m)kT}$$

$$= kJ\frac{\Diamond E_{\Delta\tau}^{Q}}{\Diamond E_{\Delta\tau}^{LB}} = kJ\ln\frac{\tau}{\Delta\tau} = \ldots = k\ln 2\,\hat{H} \quad (35)$$

$$N_{\Delta\tau}^{LB} = \Diamond E_{\Delta\tau}^{LB}/\Diamond E_{\Delta\tau_{BH}}^{LB=1} \quad (36)$$

where: 1) $q=q^Eq^Tq^Rq^v$ is the molecular partition function [2] with q^E, q^T, q^R and q^v denoting electronic, translational, rotational and vibrational motion factors (these functions are defined by (29)-(32) with the stated approximations assumed to hold for water molecules with ε_0^E denoting the electronic ground energy state assumed equal to $5kT/2$, I is the moment of inertia of molecular rotation assumed to be 2 x 10^{-47} kgm^2, σ is the symmetry number of the molecule, equal to 2 for water, and v is the vibrational frequency of water assumed equal to 1.5 x 10^9 Hz); 2) g is dimensionless and denotes the degeneracy of the ground energy state, assumed one for water; 3) T denotes the medium temperature, e.g., T=310 K for liquid water (this special medium will be used here to model that of a 70 kg individual since more than 98% of our molecules are of water which together contribute to more than 65% of our total mass); 4) m denotes the mass of a "*massive particle*" such as an atom or molecule, e.g., m=3 x 10^{-26} kg for a water molecule; 5) η is a DoF coupling constant [19] reflecting non-equilibrium thermal conditions for the medium that acts as a compression factor on the DoF of the medium at thermal equilibrium; 6) $c_V(\eta)$ is the constant volume heat capacity of the medium at its 'non-equilibrium thermal state' that yields c_V when η=1, derived from (34), with c_V corresponding to the special case when the medium is in its 'equilibrium thermal state', e.g., for our running example c_V=3 for liquid water at 310 K; 7) α is an appropriately selected constant, e.g., α=1.11 for our running example where one derives τ=102 years when η=0.66 (resulting in $c_V(\eta)$=2.49 for the heat capacity of a human, a reasonable result), and τ=42 years when η=0.6698 (resulting in $c_V(\eta)$=2.5047, a higher heat capacity value associated with an increased metabolism and decreased lifespan, and also linked to increased body heat while both T and M are kept at constant levels); 8) $c_V(\eta)kT$ denotes the energy of a theoretical thermal-energy particle (named here a "*thermote*"), e.g., $c_V(\eta)kT$=1.0657 x 10^{-20} *Joules* for our running example where $c_V(\eta)$=0.66 (as a means of comparison the energy of an electron is of 8.187 x 10^{-14} *Joules*); 9) $E=Mc^2$ is the "internal mass-energy" of the medium, e.g., for 70 *kg* of water, i.e., M=70 *kg*, one derives E=6.28 x 10^{18} *Joules* (as a means of comparison the internal energy (U) for an ideal gas model, which unlike the LTT flexible phase model does not include the medium mass-energy, is given by $U=c_VkTM/m=10^8$ *Joules* when T=1045 *K* and the c_V, M and m values are those for our running example); 10) $J=E/c_V(\eta)kT$ is the number of thermote particles in E, e.g., J=5.9032 x 10^{38} and $J^{\alpha\eta}$=2.533 x 10^{28} for our running example (where it is noted that the negative logarithmic term $J\ln(1/J^{\alpha\eta}) = -J\ln(J^{\alpha\eta})$ appearing in (28) denotes a S/k decrease reflecting the indistinguishability of J thermotes while organized in subgroups linked to atoms or molecules); 11) Q is the QoO heat energy entering the medium during $\Delta\tau$, e.g., Q=7.5825 x 10^6 *Joules* for a human consuming 1,814 *kcal* per day where $\Delta\tau$=1 day and the conversion factor of μ=4.18 *Joules/cal* is used; 12) $\Delta S=Q/T$ is the Classius entropy [14] contributed to the medium at temperature T by Q during $\Delta\tau$; e.g., ΔS=2.446 x 10^4 *Joules/K* for our example; 13) $\Delta M = Q/\Theta\,\mu$ is the QoO mass equivalent for the energy Q that is expressed as the ratio of Q to the product of Θ and μ with Θ =5,000 *kcal/kg* and μ=4.18 *Joules/cal* for our running example thus leading to ΔM=0.3628 *kg*; 14) $\Delta m=kT\ln(\tau/\Delta\tau)/\Theta\mu$ is the QoO of the mass m of a massive particle that is expressed as the ratio of the lifespan-weighted thermal-energy term $kT\ln(\tau/\Delta\tau)$ to the product of Θ and μ, e.g., Δm=2.1538 x 10^{-27} *kg* when $\Delta\tau$=1 day=1/365 year and the lifespan (τ) of the life-bits in the medium is of 102 years (as a means of comparison the mass of a hydrogen atom (m_H) is 1.6667 x 10^{-27} *kg*); 15) $\Diamond E^{Q}_{\Delta\tau} = Q$ is the QoR energy that leaves the medium during $\Delta\tau$ and is the same as the operating heat energy Q that enter it (this operation is a control or compensating action from the medium's surroundings that maintains the medium mass-energy $E=Mc^2$ constant with the passing of time); 16) $\Delta J=\Delta M/\Delta m=Q/kT\ln(\tau/\Delta\tau)$ denotes the fraction of the total number of thermotes J of the medium which equals the ratio of ΔM to Δm or equivalently the ratio of Q to $kT\ln(\tau/\Delta\tau)$, e.g., ΔJ =1.6831 x 10^{26} for our running example; 17) $\Diamond E^{LB}_{\Delta\tau} = \Delta JkT = \Diamond E^{Q}_{\Delta\tau}/\ln(\tau/\Delta\tau)$ denotes a 'life-bits (LBs) energy' fraction of the QoR radiation energy ($\Diamond E^{Q}_{\Delta\tau}$) with the fraction factor given by the reciprocal of the lifespan expression $\ln(\tau/\Delta\tau)$, e.g., $\Diamond E^{LB}_{\Delta\tau} = 7.2082 \times 10^5$ *Joules* for our running example which is 9.5% of the total emitted radiation $\Diamond E^{Q}_{\Delta\tau}$; and 18) $N^{LB}_{\Delta\tau} = \Diamond E^{LB}_{\Delta\tau} / \Diamond E^{LB=1}_{\Delta\tau_{BH}}$ denotes a "theoretical black hole based" number of life-bits that leave the medium during $\Delta\tau$, and is defined as the "QoR energy ratio" of the life-bits energy ($\Diamond E^{LB}_{\Delta\tau}$) that leave the medium during $\Delta\tau$ over the QoR energy ($\Diamond E^{LB=1}_{\Delta\tau_{BH}}$) of the single life-bit that leaves a black hole over its QoO lifespan $\Delta\tau_{BH} = 640\ln 2r_{BH}/c$ with the mass of the black hole being the same as that of the flexible phase medium, e.g., for our running example $N^{LB}_{\Delta\tau} = 64 \ x10^6$ *bits* $= 8$ *Mbytes* ($\Diamond E^{LB=1}_{\Delta\tau_{BH}}$ = 0.0112 *Joules* for this case), where it is also of interest to note that this black hole based '*QoR' energy ratio* is approximately 90% of the also black hole based '*QoO' mass ratio* $\Delta M/\Delta M_{BH}$ =71 x 10^6.

IV. Flexible Phase Versus Ideal Gas Entropies

In this section the LTT FP entropy expression in (28) for flexible phase mediums is contrasted with the non-LTT IG entropy expression for gas mediums [2], from which it was first derived [20] by applying LTT conditions.

First the LTT flexible phase medium is noted to be characterized by the following five expressions:

- The LTT flexible phase entropy (S):

$$S = k\log\Omega = kJ\ln\left(\frac{e^{5/2}}{J^{\alpha\eta}}q^{\varepsilon}q^{T}\left(q^{R}q^{v}\right)^{\frac{\eta(c_V-3/2)}{2}}\left(\frac{E}{c_V(\eta)JkT}\right)^{c_V(\eta)}\right) \quad (37)$$

- The LTT flexible phase thermal-energy (kT):

$$kT = \left(\partial(S/k)\big/\partial E\right)^{-1} = E/c_V(\eta)J \quad (38)$$

- The LTT flexible phase constant volume heat capacity ($c_V(\eta)$):

$$c_V(\eta) = \frac{1}{J}\frac{\partial E}{\partial(kT)} = \frac{kT}{J}\frac{\partial(S/k)}{\partial(kT)} = 3/2 + \eta(c_V - 3/2) \quad (39)$$

- The LTT flexible phase law:

$$VP = JkT = E/c_V(\eta) \quad (40)$$

- The LTT flexible phase number of particles:

$$J = E/c_V(\eta)kT = M/(c_V(\eta)kT/c^2) \quad (41)$$

where all the variables in (37)-(41) were earlier defined for (28) except for P in (40), the pressure on the FP medium.

Secondly the non-LTT ideal gas medium is noted to be characterized by the following five expressions:

- The non-LTT ideal gas Boltzmann entropy (S_{IG}):

$$S_{IG} = k\log\Omega_{IG} = kJ_{IG}\ln\left(\frac{e^{5/2}}{J_{IG}}q^{\varepsilon}q^{T}q^{R}q^{v}\left(\frac{U}{c_V J_{IG}kT_{IG}}\right)^{c_V}\right) \quad (42)$$

- The non-LTT ideal gas thermal-energy (kT_{IG}):

$$kT_{IG} = \left(\partial(S_{IG}/k)\big/\partial U\right)^{-1} = U/c_V J_{IG} \quad (43)$$

- The non-LTT ideal gas constant volume heat capacity (c_V):

$$c_V = \frac{1}{J_{IG}}\frac{\partial U}{\partial(kT_{IG})} = \frac{kT_{IG}}{J_{IG}}\frac{\partial(S_{IG}/k)}{\partial(kT_{IG})} = c_V \quad (44)$$

- The non-LTT ideal gas law:

$$V_{IG}P_{IG} = J_{IG}kT_{IG} = U/c_V \quad (45)$$

- The non-LTT ideal gas number of particles:

$$J_{IG} = U / c_V k T_{IG} = M / m \tag{46}$$

where a comparison of (37)-(41) and (42)-(46) reveals at least six basic differences between them. They are:

1) The LTT internal energy (E) in (37) is the "*regulated*" total energy of the FP medium, thus it includes all gravitational/non-gravitational interactions of its particles, while the non-LTT internal energy (U) in (42) is the partial energy of the IG medium that only includes the kinetic-energy/potential-energy of its particles.
2) The LTT number of particles (J) in (37) refers to thermotes of the FP medium, while the non-LTT number of particles (J_{IG}) in (42) refers to "massive particles" of the IG medium.
3) The LTT pressure (P) in (40) refers to the pressure exerted on the FP medium volume by all the thermotes making up the internal energy E, while the non-LTT pressure (P_{IG}) in (45) refers to the pressure exerted on the IG medium by all the massive particles making up the internal energy U.
4) The LTT DoF coupling constant η in (39) acts as a 'compression factor', since $\eta < 1$, on the heat capacity c_V or DoF=2c_V of the medium that conveys the actual non-equilibrium thermal conditions, while the non-LTT ideal-gas entropy model of (42) does not have such constant.
5) The number of LTT microstates (Ω) in (37) is exponentially related to the number of thermotes (J), while the number of non-LTT microstates (Ω_{IG}) in (42) is exponentially related to the number of massive particles (J_{IG}). As a result, the LTT Boltzmann entropy (S) is expected under most conditions to be significantly larger than the non-LTT Boltzmann entropy (S_{IG}) since $J >> J_{IG}$.
6) The FP entropy (37) satisfies the constant total medium energy $E = Mc^2 = \sum_{i=1}^{r} n_i \varepsilon_i$ and constant number of thermotes $J = \sum_{i=1}^{r} n_i$ constraints with n_i being the number of thermotes with the *i-th* microstate energy ε_i of r possible, while the IG entropy (42) satisfies the constant internal energy $U = \sum_{i=1}^{r} \varsigma_i \varepsilon_i$ and constant number of atoms/molecules $J_{IG} = \sum_{i=1}^{r} \varsigma_i$ constraints with ς_i being the number of atoms/molecules with the *i-th* microstate energy ε_i of r possible.

V. HUMAN LIFESPAN STUDY

The flexible phase ULTE (or FP ULTE) is now illustrated with a human lifespan study. The flexible phase medium assumed for this case is well suited to model the human medium since it directly reflects either atom or molecular interactions of both gravitational and non-gravitational origin. Before the flexible phase ULTE was first investigated in [20], the source entropy model for an ideal gas had been used by the author in [1] since it led to simple tractable solutions. Yet this solution was deficient in its applicability since an IG entropy model was being applied to a non-gas liquid medium. Fortunately this deficiency has now been addressed by the discovery of the flexible phase ULTE as is illustrated next. First it is noted that (28) and (33) as well as the mass density equation for a water medium given by $V=M/1000=E/1000c^2$, lead to the following *DoF lifespan equation* and specific result of $\tau = 102$ *yrs* for a 70 *kg* individual whose non-equilibrium thermal state corresponds to a DoF coupling constant specified by η=0.66:

$$\tau = \Delta\tau \frac{e^{5/2}}{J^{\alpha\eta}} q^E q^T \left(q^R q^v\right)^{\frac{\eta(c_V - 3/2)}{2}} \approx 102 \text{ yrs} \tag{47}$$

where for our running example α=1.11, q^E=0.0821, q^E=5.49x10^{30}, q^R=7.6749, q^v=4.3x10^3, $\Delta\tau$=1/365 years and c_V=3. The $\tau = 102$ *yrs* result of (47) is then noted to coincide with the following *NCR lifespan equation* result (used in [22]-[23] to derive a life expectancy premium):

$$\tau = \Delta\tau \left(\frac{M}{\Delta M}\right)^2 \approx 102 \text{ yrs} \tag{48}$$

that surfaces from the ULTE (9), and is illustrated with M=70 *kg* and ΔM=0.3628 *kg* of food/day (corresponding to a 1,841 *kcal/day* diet).

The two different approaches to the evaluation of τ expressed by (47) and (48) have resulted in identical results when sensible assumptions for the individual's medium were made. Moreover, it is noticed that (47) and (48) can be equated to yield the following *mathematical micro DoF* (reflected by the η value) to *physical macro NCR* (reflected by the ΔM value) non-linear relationship:

$$\Delta M = M \sqrt{\frac{J^{\alpha\eta}}{e^{5/2}} \left(q^\varepsilon q^T\right)^{-1} \left(q^R q^v\right)^{-\frac{\eta(c_V - 3/2)}{2}}} \tag{49}$$

It is further noticed that if one assumes that the childhood lifespan of the 70 *kg* individual is of 18 years, his/her expected total lifespan would be of 120 years (this number of years is close to the maximum recorded lifespan for a human which exceeds 122 years [24]). Moreover, lower adult lifespans would be found if the daily consumption of food is greater than 0.3628 *kg* or alternatively if the DoF coupling constant η increases in value from 0.66. For instance, when ΔM=0.5654 *kg* for a 2,827 *kcal/day* diet or η=0.6698 one derives a theoretical adult lifespan of 42 years from both (47) and (48). The decrease in adult lifespan from 102 to 42 years is substantial but expected since the 70 *kg* individual has significantly increased his/her ΔM (or increased the value of η) while still maintaining the same mass of 70 *kg*. Clearly this implies an increased metabolic strain per day leading to the individual aging at a faster rate [18].

The above results have posited the theoretical DoF lifespan equation method of (47) as a sensible alternative to the theoretical NCR lifespan equation method of (48) (both linked through (49)), for the study of biological lifespan.

VI. Summary and Conclusions

This paper described the revelation of a novel thermodynamics entropy method for mediums with an arbitrary phase. The method surfaced naturally from linger thermo theory, one of two major designs of the UC duality, the other is latency information theory. The UC duality was first identified in 1978 in LQG control and then applied to quantized control to yield a technique called Matched Processors. This control technique was the time-certainty dual of the space-uncertainty Matched Filters for bit detection. Moreover, it advanced a practical parallel processing approach to quantized control that did not suffer of what Bellman called "the curse of dimensionality" of his Dynamic Programming. More than two decades later DARPA funded research on high-performance KA adaptive radar ignited the discovery of the time dual for information theory that was named latency theory and their synergistic unification that was named latency information theory or LIT. From LIT a fast and powerful approach to adaptive radar surfaced which was named power centroid radar or PC radar. In turn, this discovery eventually led to the revelation of the time dual for thermodynamics that was named lingerdynamics as well as their synergistic unification that was named linger thermo theory or LTT. Finally from LTT the entropy solution to flexible phase mediums of this paper surfaced. More specifically the FP entropy for flexible mediums was found to arise from the expression for the entropy of an ideal gas when adapted to satisfy a LTT mass-energy regulation constraint. The reasonableness of the derived LTT FP entropy scheme in modeling the entropy of a biological medium was verified when its predictions for the theoretical adult lifespan of an individual were found to match those earlier derived using an alternative LTT nutritional consumption rate or NCR method. Moreover, it was found that the FP entropy method described the control of lifespan from a novel *micro* DoF metric perspective not offered by the earlier *macro* NCR metric approach. These results have established the DoF based FP entropy approach as a sensible tool for the study of the biophysical chemistry of lifespan. In the future it is expected that LTT FP entropy method will find broad use in lifespan investigations since gravitational/non-gravitational interactions of atoms or molecules are the rule rather than the exception. Finally, researchers working in their own challenging problems, regardless of field of interest, are encouraged to reflect on the UC duality counsel that says, "Synergistic physical and mathematical dualities naturally arise in efficient system designs."

Dedication

In this the 120th year since his birth this article is dedicated to the memory of Norbert Wiener, originator of cybernetics.

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