Power Centroid Radar and its Rise from the Universal Cybernetics Duality

Erlan H. Feria
Department of Engineering Science and Physics
The College of Staten Island of the City University of New York

ABSTRACT
Power centroid radar (PC-Radar) is a fast and powerful adaptive radar scheme that naturally surfaced from the recent discovery of the time-dual for information theory which has been named “latency theory.” Latency theory itself was born from the universal cybernetics duality (UC-Duality), first identified in the late 1970s, that has also delivered a time dual for thermodynamics that has been named “lingerdynamics” and anchors an emerging lifespan theory for biological systems. In this paper the rise of PC-Radar from the UC-Duality is described. The development of PC-Radar, US patented, started with Defense Advanced Research Projects Agency (DARPA) funded research on knowledge-aided (KA) adaptive radar of the last decade. The outstanding signal to interference plus noise ratio (SINR) performance of PC-Radar under severely taxing environmental disturbances will be established. More specifically, it will be seen that the SINR performance of PC-Radar, either KA or knowledge-unaided (KU), approximates that of an optimum KA radar scheme. The explanation for this remarkable result is that PC-Radar inherently arises from the UC-Duality, which advances a “first principles” duality guidance theory for the derivation of synergistic storage-space/computational-time compression solutions. Real-world synthetic aperture radar (SAR) images will be used as prior-knowledge to illustrate these results.

Keywords: Radar, SAR, Latency Information Theory, Universal Cybernetics Duality, Linger Thermo Theory, Biophysics, Lifespan

I. INTRODUCTION
Radar uses radio waves to find the range, altitude, direction, or speed of objects. Its applications are widespread such as in defense, space, commercial and medical investigations of tissue, heart and respiratory states. In demanding applications such as in moving target indicator (MTI) radar, for ground or airborne targets, its performance can be significantly degraded by interference and thermal white noise. The interference can be of various kinds such as clutter, jammer, range walk, channel mismatch, internal clutter motion and antenna array misalignments. Thus in these applications adaptive radar systems are designed that address any changes that may occur in the operating environment [1]. Of the aforementioned interference types clutter, which are returns from the range-bin where a target is being investigated, is without doubt the one most difficult to adapt to.

To attend to the clutter problem two basic approaches are used in adaptive radar. One approach is knowledge-unaided (KU), i.e., prior-knowledge about the clutter is not used, that leads to simple radar schemes but with a marginal SINR performance, and the other is knowledge-aided (KA) that leads to a superior SINR radar performance but with significant implementation complexities. The standard KU radar scheme is sample covariance matrix radar (SCM-Radar). In KU SCM-Radar clutter samples are used from the range-bin where a target is being investigated, as well as its close-by neighbors, to find the clutter covariance matrix. Unfortunately, however, the SINR performance derived with KU SCM-Radar is often marginal since it is only satisfactory when the clutter has stationary statistics, which is seldom the case. The second KA approach uses prior-knowledge such as synthetic aperture radar (SAR) imagery of range-bin locations that the radar system may investigate. KA radar techniques of this kind were developed, for instance, under the 2001-2005 Defense Advanced Research Projects Agency (DARPA) KA Sensory Signal Processing and Expert Reasoning (KASSPER) program [2] (the author joined the KASSPER program at its tail end to help with the compression of the SAR imagery being used). The KASSPER schemes were applied to ground moving target indicator (GMTI) applications. Although some of the radar schemes could yield a superior SINR radar performance their designs were subjected to severe constraints. One was the daunting storage-space needs of SAR-imagery and another was the extreme computational-time burden of on-line clutter-covariance matrix evaluations. Another important limitation was the absence of a “first-principles” guiding theory for radar designs that would inherently lead to fast and powerful synergistic storage-space/computational-time compression solutions.
After the author joined the KASSPER project in its last year, with his assigned task being the efficient predictive-transform (PT) compression of the SAR imagery for radar use [2], he soon discovered that the radar schemes that had been developed could not be radar blind [3], which clearly complicated the radar system implementations. In turn this realization led him to search for a guiding theory for radar design in the universal cybernetics duality (UC-Duality), hypothesized by him in the late 1970s while pursuing graduate studies in cybernetics [4]. Soon after the start of this search a fast and powerful synergistic storage-space/computational-time compression radar solution surfaced. This solution was power-centroid (PC) radar (PC-Radar) that is US patented [5]-[6], whose SINR performance emulates that of an optimum scheme, referred here as Optimum-Radar, that uses covariance matrix tapers [7] to model the interference plus noise covariance. PC-Radar was at first KA [2]. Then in 2009, four years after the end of the KASSPER program, a knowledge-unaided (KU) version was first offered [8]. This KU version of PC-Radar like the previous KA version was found to yield a superior SINR performance, a remarkable result. The emergence of PC-radar from the UC-Duality, see Fig. 1, is briefly reviewed next.

In 1978 from investigations of Linear Quadratic Gaussian Control (LQG-Control) [9], the UC-Duality hypothesis surfaced. The UC-Duality revelation was that,

“Synergistic physical/mathematical dualities naturally arise in efficient system designs”

More specifically, the “physical duality” conveyed the separation of the system design into a space-uncertainty communication problem and a time-certainty control problem, while the “mathematical duality” conveyed the appearance of identical mathematical structures in the separately designed communication/control subsystems. In turn this revelation led to “Matched Processors (MPs) for Optimum Control”, the author’s 1981 Ph.D. [10]. While LQG-Control dealt with continuous control, MPs-Control dealt with quantized control. In MPs-Control the certainty-based parallel structures of the Matched Processors controller [11] was the control’s certainty-based dual of communication’s uncertainty-based parallel structures of Matched-Filters for bit detection [12]. A remarkable result [11] of MPs-Control was that unlike Bellman’s Dynamic Programming [13], it did not suffer of what Bellman called “the curse of dimensionality” when referring to the exponential increase in computational burden as the process state dimension increased in value.

Fig. 1 The Universal Cybernetics Duality (UC-Duality)
More than two decades after Matched Processors first surfaced, KA adaptive radar design issues served as the catalyst to the discovery of Latency Theory and Lingerdynamics Theory as time-certainty duals for space-uncertainty Information Theory [14] and Thermodynamics Theory [15], respectively. In turn, these theories led to the synergistic Latency Information Theory (LIT) [16] and Linger Thermo Theory (LTT) [17]. These two theories addressed four different types of system functions. The four were: 1) a “source” uncertainty function measured by a source entropy space-metric (this metric is the Shannon’s “info-source” entropy [14] in LIT and the Boltzmann’s “thermo-source” entropy [15] in LTT); 2) a “processor” certainty function measured by a novel processor entropy time-metric; 3) a “retainer” uncertainty function measured by a novel retainer entropy space-metric; and 4) a “mover” certainty function measured by a novel mover entropy time-metric. Yet the nature of the LIT and LTT space/time metrics were quite different. In the case of LIT they were time invariant, or stationary in nature, while in the LTT case they were time varying, or dynamic in nature. The LTT dynamic property has roots in one of the four laws of thermodynamics that drive the universe [15], or more specifically, the 2nd law of thermodynamics that states that the Boltzmann entropy (or equivalently the thermo-source entropy space-metric) increases with time for a closed system. It can be shown that similar increases occur to the remaining LTT space/time metrics with the passing of time [17]. In Appendix C a brief outline for LTT is given where the basic ideas are illustrated with black-hole, photon-gas and flexible-phase mediums. Moreover, for the flexible-phase medium it is shown that an entropy theory inherently emerges from LTT that in a sensible and compelling manner independently supports a previously offered LTT lifespan theory for biological systems [17].

Next we describe the rise of PC-Radar from the stationary Latency Information Theory or LIT. PC-Radar emerged from Latency Theory’s Processor Coding, see top of Fig. 1, which is the time dual of Information Theory’s Source Coding. While a source coder such as a predictive transform (PT) one, aims for high “bit-space compressions”, a processor coder such as a power centroid (PC) one, aims for high “bor-time compressions”, i.e., the smallest possible number of binary operator (or bor) levels from processor input to output.

In PC-Radar a processor encoder followed by a processor decoder derives the front clutter covariance matrix \( C_f(\theta_{AAM}) \), that is also a function of any existing antenna array misalignment angle \( \theta_{AAM} \) as noted in Section II. The objective of the encoder is to measure the power-centroid or PC of the clutter emanating from the front range-bin displayed in Fig. 2. This figure shows the physical antenna pattern (PAP) for a phased array antenna of a moving target indicator or MTI that is directed towards an investigated target on the front range-bin. All the terms in Fig. 2 are defined in Section II. In a symmetrical stationary-clutter scenario the range-bin PC location would be the same as where the PAP points.

**Fig. 2 The Overall MTI Radar System and its Performance Metric**
On the other hand, the objective of the decoder is to use the measured PC to select a \( C'_f(\theta_{AAM}) \) realization from a stored set, where the elements of the set are evaluated off-line and are matched to unique range-bin PC and \( \theta_{AAM} \) quantization levels. The best matched \( C'_f(\theta_{AAM}) \) is then used in an interference plus noise covariance \((R)\) expression, see the bottom of Fig. 2, that is modeled with covariance-matrix-tapers \([7]\) that contains the other back-clutter \((C'_b(\theta_{AAM}))\), jammer \((C_j)\), range-walk \((C_{RW})\), internal-clutter-motion \((C_{ICM})\), channel-mismatch \((C_{CM})\) and thermal-noise \((C_n)\) covariance cases (in Sections II and Appendix A the mathematical models for these much easier to find covariances are reviewed). As is seen later in Section II the inverse of \( R \) is then multiplied by the target steering vector \((s)\) to yield the radar weighting gain \((w)\) resulting in a superior SINR radar performance.

Both KA and KU PC-Radar are found to emulate the SINR performance of Optimum-Radar. The explanation for this exceptional result is the use of “mathematical antenna patterns” (MAPs) in the off-line evaluation of the \( C'_f(\theta_{AAM}) \) set. More specifically, when \( C'_f(\theta_{AAM}) \) is evaluated off-line, the PAP appearing in its covariance matrix definition is replaced with a MAP that points towards its matched PC value. In this way the MAP acts as a control compensator for non-stationary clutter PC measurements that deviate from the PAP pointing direction. When the PC processor-decoder receives a quantized PC, either from KA or KU measurements, also an appropriate \( \theta_{AAM} \) level, it then retrieves from its memory the \( C'_f(\theta_{AAM}) \) case that matches them.

Under severely taxing environmental conditions it will be seen later in Section IV that PC-Radar yields outstanding SINR results, even if only a few quantization levels are used for the PC. For the KA case it also offers a significant implementation advantage since “radar-blind” image compression of SAR imagery is now possible. Moreover, in the KU case the power centroid is derived directly from a sample covariance matrix. This result is remarkable since with a very simple KU scheme PC-Radar approaches the SINR performance of Optimum-Radar.

In the future the exposed synergistic PC-Radar ideas of this paper are expected to find extensive use in fields such as smart antennas, with applications not only found in radar but also in radio communications where non-stationary clutter interferences are the rule rather than the exception as the complexity and demands of wireless multi-media networks continuously increase. Finally researchers working in their own complex problems, in any field, should take note that the UC-Duality may possibly guide them towards the best synergistic solutions.

The paper organization is as follows. In Section II the mathematical statement of the MTI radar problem is given. In Section III the PC-Radar methodology is outlined. In Section IV simulation results are given that contrast the SINR performance of PC Radar with that of SCM-Radar and Optimum-Radar. The paper ends with a summary and conclusions.

II. THE MTI RADAR PROBLEM

As noted earlier the key to a superior SINR radar performance while subjected to intense interference plus noise disturbances is to appropriately adjust the radar system parameters as the disturbance characteristics change with location. A system requiring such adaptation is the MTI radar system of Fig. 2. The statement of the radar problem for a MTI radar system is presented in this section in seven subsections starting with the six interference types previously alluded to.

A. Six Interference Types

1) ‘Range Walk (RW)’. This type of interference is due to the movement of the radar platform during a coherent pulse interval (CPI). The CPI denotes the time delay associated with the transmission of \( M \) pulses by \( N \) antenna elements of the “phased array antenna” assumed in our MTI radar model. The product of \( N \) and \( M \), i.e., \( NM \), represents the degrees of freedom (DoF) of the radar system. This number is also the assumed number of cells for the investigated range-bin displayed in Fig. 2. The covariance matrix for range walk is studied in Appendix A where expressions (A.9)-(A.12) define it.

2) Clutter (c): These are antenna gain weighted returns from the range-bin where the appearance of a target is being investigated at boresight as seen in Fig. 2. For the physical antenna pattern or PAP emerging from \( N \) antenna elements the gain is designed to be high for the front-lobes and low for the back-lobes. Thus the clutter collected by the radar from the back range-bin cells is insignificant when compared to that collected from the front range-bin cells. The covariance matrix for the front clutter will be studied later with expressions (18)-(26) defining it.
3) **Jammer (J):** These are emissions emanating from range-bin cells that attempt to disrupt radar searches. The covariance matrix for jammer is studied in Appendix A where expressions (A.2)-(A.8) define it.

4) **Internal Clutter Motion (ICM):** This interference describes a change in the range-bin clutter that may occur during the CPI. The covariance matrix for internal clutter motion is studied in Appendix A where expressions (A.13)-(A.16) define it.

5) **Channel Mismatch (CM):** These are signal channel mismatches whose origin can be ‘angle dependent’, ‘angle independent narrowband’ and ‘finite bandwidth’. The covariance matrix for channel mismatch is studied in Appendix A where expressions (A.17)-(A.29) define it.

6) **Antenna Array Misalignment Angle (θ_{AAM}):** This is an antenna misalignment angle whose value impacts the evaluation of the steering vectors linked to each range-bin cell. Next we take a close look at all the major radar signals.

### B. Major Radar Signals

There are two major complex signals that are received by the MTI, with both being NM dimensional. One signal is the normalized steering vector of the target of interest (s) and the other is the interference plus noise vector (x). These two signals are added up to form the total received signal (r) defined according to:

\[ r = x + s \] (1)

It is then the task of the MTI to multiply this received signal by a complex weighting vector (w) of dimension NM to yield a scalar output (y) whose value is then used to determine if a target has been received or not. Thus one derives

\[ y = w^H r = w^H(x + s) = w^Hx + w^Hs \] (2)

where \( w^Hx \) is the signal contribution and \( w^Hs \) is the interference plus noise contribution to \( y \). Note that ‘\(^H\)’ denotes a vector complex conjugate transposition, i.e., a Hermitian transpose. The derivation of the gain \( w \) will be discussed next.

### C. The Performance Metric

In the radar system design one aims to find an expression for \( w \) that maximizes the ratio of the signal power \( |w^Hs|^2 \) to the expected interference plus noise power \( E[w^Hxx^HW] = w^HE[xx^H]w = w^HRw \). In this way the relative signal contribution to the amplitude of \( y \) will be the largest possible when a target appears on the investigated range-bin location. Thus our objective is then to maximize the SINR expression given by

\[ \text{SINR} = \frac{|w^Hs|^2}{w^HRw} \] (3)

with \( R \) denoting a complex NM x NM interference plus noise covariance matrix. The maximization of (3) then results in the well known Wiener-Hopf equation for the optimum gain (\( w^* \)) expression according to:

\[ w^* = R^{-1}s \] (5)

Associated with (5) one then derives the optimum SINR (SINR\(^*\)) according to:

\[ \text{SINR}^* = s^HR^{-1}s \] (6)

Next we discuss the covariance matrix tapers model used for the interference plus noise covariance matrix (R).

### D. The Interference Plus Noise Covariance Model:

The study of the aforementioned six interference cases in the context of R yields the following covariance matrix tapers model for R [7]:

\[ R = (C^c_\theta(\theta_{AAM}) + C^b_\theta(\theta_{AAM})) O (C_{RW} + C_{ICM} + C_{CM} + C_J O C_{CM} + C_n \] (7)

\[ C_{CM} = C_{AD} + C_{AIN} + C_{FB} \] (8)

where: 1) \( C^c_\theta(\theta_{AAM}) \) and \( C^b_\theta(\theta_{AAM}) \) are complex NM x NM front and back clutter covariances that are functions of the antenna array misalignment angle \( \theta_{AAM} \); 2) \( C_{AD}, C_{AIN}, C_{FB} \) are composite and complex NM x NM “angle dependent (AD)”, “angle independent narrowband (AIN)” and “finite bandwidth (FB)” channel mismatch covariances, respectively, that are added to yield the total channel mismatch covariance \( C_{CM} \), see Appendix A; 3) \( C_{RW} \) is a complex NM x NM range walk covariance, see Appendix A; 4) \( C_{ICM} \) is a complex NM x NM internal
clutter motion covariance, see Appendix A; 5) $C_n$ is a $NM \times NM$ thermal noise covariance, see Appendix A; and 6) the symbol “$\mathbf{O}$” denotes Hadamard term by term products of the elements of two matrices.

The mathematical expressions defining the target steering vector ($s$), the antenna pattern for a uniform linear array (ULA) and the front clutter covariance matrix $C_f(\theta_{ULA})$ are given next. As noted earlier the definition for the remaining covariances in the $R$ expressions (7)-(8) are as defined in Appendix A. In our later simulations the values for these matrices are assumed to be either known or of zero value as is the case for the back clutter covariance matrix $C_b(\theta_{ULA})$. Next the mathematical model for the target signal is noted.

### E. The Target Return

The MTI system is assumed to receive from the target a normalized steering vector ($s$). This signal is complex, $MN$ dimensional and is defined according to:

$$ s = [s_1(\theta), s_2(\theta), \ldots, s_M(\theta)]^T / \sqrt{NM} $$

$$ s_k(\theta) = e^{i2\pi(k-1)\theta} s_k(\theta) \quad \text{for} \ k = 1, \ldots, M \quad \text{(9)} $$

$$ s_k(\theta) = [s_1(\theta), s_2(\theta), \ldots, s_M(\theta)] \quad \text{(10)} $$

$$ s_k(\theta) = e^{i2\pi(k-1)\theta} \quad \text{for} \ k = 1, \ldots, N \quad \text{(11)} $$

$$ f_p = f_c / f_r \quad \text{(12)} $$

$$ f_c = 2\nu / \lambda = 2(\nu / c) f_r \quad \text{(13)} $$

$$ f_r = 1 / T_r \quad \text{(14)} $$

$$ \theta_i = (d / \lambda) \sin(\theta) \quad \text{(15)} $$

where: 1) $\theta$ is the value of the boresight angle ($\theta$) where the target resides, $\theta = 0^\circ$ for the case displayed in Fig. 2; 2) $f_c$ is the carrier (or operating) frequency of the radar system; 3) $d$ is the antenna inter-element spacing; 4) $\lambda$ is the operating wavelength; 5) $\theta_i$ is the normalized $\theta_i$; 6) $T_r$ is the pulse repetition interval (PRI); 7) $f_r$ is the pulse repetition frequency (PRF); 8) $v$ is the target radial velocity; 9) $c$ is the speed of light; 10) $f^r_D$ is the target Doppler; and 11) $f_D^r$ is the normalized Doppler. Next the antenna pattern associated with the $N$ antenna elements is studied.

### F. The Antenna Pattern

The MTI is characterized by a uniform linear array (ULA) that yields the following analytical and normalized gain expression for an antenna pattern with $NM$ degrees of freedom:

$$ g^H_i = K^f \left| \frac{\sin \left( d \left( \frac{1}{\lambda} \sin(\theta) \right) \right)}{\sin \left( \frac{d}{\lambda} \left( \sin(\theta) - \sin(\theta_i) \right) \right) / NM} \right| \quad \text{for} \ i = 1, \ldots, NM \quad \text{(17)} $$

where: 1) $\theta$ denotes the boresight angle; 2) $\theta^j$ is the value of the boresight angle corresponding to the $i^{th}$ range-bin cell; 3) $\theta_i$ is the value of the boresight angle where the target of interest resides; 4) $N$ is the number of antenna elements; 5) $M$ is the number of pulses transmitted during the coherent pulse interval; 6) $NM$ is the number of range-bin cells which is the same as the number of degrees of freedom; 7) $\lambda$ is the operating wavelength; and 8) $K^f$ is the front antenna gain constant. Next the mathematical expression for the front clutter covariance matrix that the adaptive radar must evaluate on-line is described.

### G. The Front Clutter Covariance Matrix:

The front clutter covariance matrix ($C_f(\theta_{ULA})$) is modeled according to:

$$ C_f(\theta_{ULA}) = \sum_{i=1}^{NM} x_i g^H_i c_i(\theta_{ULA}) c_i^H(\theta_{ULA}) $$

where:
1) \( \{ x_i : i = 1, ..., NM \} \) are the clutter powers of the front range-bin where \( x_i \) denotes the \( i \)th cell clutter power.

2) \( \{ g_i^\theta : i = 1, ..., NM \} \) are the \( NM \) gains of the antenna pattern (17) that points towards the target boresight angle \( \theta \).

3) \( \{ x_i g_i^\theta : i = 1, ..., NM \} \) denotes the \( NM \) antenna gain modulated clutter powers of the front range-bin.

4) \( \{ c_i(\theta_{\text{AAM}}) : i = 1, ..., NM \} \) denotes the \( NM \) steering vectors of the front range-bin cells whose values depend on the antenna array misalignment angle \( \theta_{\text{AAM}} \). In particular, \( c_i(\theta_{\text{AAM}}) \) denotes the \( i \)th cell steering vector, which is complex and \( MN \) dimensional, defined according to:

\[
\begin{align*}
\mathbf{c}_i(\theta_{\text{AAM}}) &= \left[ f \mathbf{z}_1(\theta', \theta_{\text{AAM}}) \; f \mathbf{z}_2(\theta', \theta_{\text{AAM}}) \; \ldots \; f \mathbf{z}_M(\theta', \theta_{\text{AAM}}) \right]^T \\
\mathbf{z}_k(\theta') &= e^{j2\pi(k-1)\bar{\theta}'} \\
c_{ki}(\theta') &= e^{j2\pi(k-1)\bar{\theta}'} \\
\bar{\theta}' &= (d/2)\sin(\theta + \theta_{\text{AAM}})
\end{align*}
\]

where 1) \( v_p \) is the radar platform speed; 2) \( \bar{\theta}' \) is the normalized \( \theta' \), inclusive of the antenna array misalignment angle \( \theta_{\text{AAM}} \) as seen from (25); and 3) \( \beta \) is the ratio of the distance traversed by the radar platform during the PRI, i.e., \( v_p T_r \), to the half antenna inter-element spacing, \( d/2 \). The remaining parameters for expressions (19)-(25) were earlier defined for (9)-(16). Finally, it is noted that the first element of \( \mathbf{c}_i(\theta_{\text{AAM}}) \) divided by the variance of the thermal white noise \( \sigma_n^2 \) defines the front clutter to noise ratio (CNR\( f \)), i.e.,

\[
\text{CNR}^f = C'_f(1,1)/\sigma_n^2
\]

5) \( \{ c_i(\theta_{\text{AAM}})e_i^\alpha(\theta_{\text{AAM}}) : i = 1, ..., NM \} \) denotes the set of cell steering matrices.

III. POWER CENTROID RADAR

The main goal of a PC-Radar scheme is the adaptive evaluation of the front clutter covariance matrix \( C'_f(\theta_{\text{AAM}}) \) (18) for later use in determining the interference plus noise covariance \( R \) expression (7)-(8), where it is also assumed that the remaining covariances in the expressions can be independently found. In descending order of storage-space/computational-time complexity there are four PC-Radar schemes. Two are KA and two are KU. Each is described next:

A. Knowledge Aided PC-Radar:

In KA PC-Radar the front clutter covariance matrix \( C'_f(\theta_{\text{AAM}}) \) of (18) is replaced by a KA version \( \hat{K}_i C'_f(\theta_{\text{AAM}}) \) defined according to:

\[
\hat{K}_i C'_f(\theta_{\text{AAM}}) = \eta \sum_{i=1}^{NM} g_i^\theta e_{ki}(\theta_{\text{AAM}}) e_i(\theta_{\text{AAM}})
\]

\[
1 \leq \text{PC}_{ki} = \sum_{i=1}^{NM} \bar{\xi}_i g_i^\theta e_i^\alpha / \leq NM
\]

and

\[
P = \sum_{i=1}^{NM} \bar{\xi}_i g_i^\theta e_i^\alpha
\]
where: 1) \( \{ \tilde{x}_i : i=1, \ldots, NM \} \) are the clutter powers of the range-bin extracted from SAR imagery; 2) \( \{ g_1^{\theta=0^\circ}, \ldots, g_{NM}^{\theta=0^\circ} \} \) denotes the physical antenna pattern or PAP directed towards the target at the zero boresight angle; 3) \( P \) denotes the received total clutter power; 4) \( PC_{KA} \) denotes the \( KA \) power centroid of the range-bin; 5) \( \{ g_1^{\theta=0^\circ}, \ldots, g_{NM}^{\theta=0^\circ} \} \) denotes the mathematical antenna pattern or MAP derived from (17) when it is directed towards the boresight angle \( \theta_{KA} \) corresponding to the range-bin location of \( PC_{KA} \) rather than the boresight angle of \( 0^\circ \) where the target is being investigated in our running example. In the disturbance control (or cybernetics) solution to the clutter covariance modeling of (27), the MAP acts as a “compensating vector gain” for the non-stationary clutter that disturbs the “stationary clutter” associated with (18), which yields an optimum result when both the location of the target and the PC are the same; 6) \( \{ c_i(\theta_{AAM}) : c_i(\theta_{AAM}) : i=1, \ldots, NM \} \) denotes the set of cell steering matrices for the range-bin; and 7) \( \eta \) is a normalizing clutter power constant.

In Fig. 3 a block-diagram description for the computations of (27)-(29) is shown. The block-diagram is noted to have two major components. One is a processor-encoder that derives \( PC_{KA} \) from range-bin measurements which are extracted from SAR imagery. The second major component is a processor-decoder that receives \( PC_{KA} \) and appropriately derived values for \( \eta \) and \( \theta_{AAM} \) to yield \( ^K C_i(\theta_{AAM}) \). As seen from the top of Fig. 1 this processor encoder/decoder structure is the computational-time compression dual of the source encoder/decoder structure for storage-space compression, and thus transparently displays its UC-Duality roots. From (27) it is apparent that its on-line computational-time burden is significant due to the high dimensionality of all the needed operations. However, this problem can be greatly alleviated if one restricts the possible values that \( PC_{KA} \) as well as \( \theta_{AAM} \) may have, thus naturally leading us to exceedingly fast memory fetches based implementations. This approach gives rise to the second type of KA PC-Radar with power centroid or PC quantizations, called here QKA PC-Radar, that is described next.

**B. Knowledge Aided PC-Radar with \( PC_{KA} \) quantization**

In QKA PC-Radar the front clutter covariance matrix \( ^K C_i(\theta_{AAM}) \) of (27) is replaced by its PC quantized version \( ^{QK} C_i(\theta_{AAM}) \) defined according to:

\[
^{QK} C_i(\theta_{AAM}) = \eta \sum_{i=1}^{NM} g_i^{\theta=0^\circ} c_i(\theta_{AAM}) c_i^H(\theta_{AAM})
\]

where expressions (27) and (30) are the same except that \( PC_{KA} \) in (27) is replaced with \( Q(PC_{KA}) \) to yield (30). The quantizer leading to \( Q(PC_{KA}) \) can be defined, for instance, as follows:

\[
1 \leq Q(PC_{KA}) = \min_i|PC_{KA} - i| \leq NM,
\]

\[
 i \in \left\{ 1 + \frac{NM - 1}{L + 1}, 1 + 2 \frac{NM - 1}{L + 1}, \ldots, 1 + \frac{NM - 1}{L + 1} \right\}
\]

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where the value of $L$ denotes the number of quantization levels. The quantization levels for $PC_{K_A}$ are uniformly distributed over the range-bin according to (31).

In Fig. 4 a block-diagram description for the on-line computational structure that determines (30) is given. The processor encoder for this case performs the evaluation of $PC_{K_A}$ according to (28) and then follows it by the PC quantization according to (31) thus yielding as output $Q[PC_{K_A}]$. On the other hand, the processor decoder is a fast lookup memory that stores off-line derived evaluations of (30) corresponding to allowed quantization levels for $PC_{K_A}$ and known values for $\eta$ and $\theta_{AAM}$. It is also assumed here that appropriate quantizations of $\eta$ and $\theta_{AAM}$ are available for off-line evaluations of $(g,C_i(\theta_{AAM}))$. In this way the processor decoder selects from its memory the case that is matched to the received $Q[PC_{K_A}]$ as well as on-line determined quantizations of $\eta$ and $\theta_{AAM}$. Moreover, it is noted that storage-space savings can be achieved by increasing the computational-time burden. For instance, this occurs when quantization versions of $(g,C_i(\theta_{AAM}))/\eta$ rather than of $(g,C_i(\theta_{AAM}))$ are saved, with the best case of $(g,C_i(\theta_{AAM}))/\eta$ first fetched from the memory and then multiplied by the on-line evaluated $\eta$. At this point our attention turns to the prior-knowledge, i.e., the SAR imagery. It was noted earlier that using a radar-blind storage-space compression approach one can drastically compress the SAR images while still yielding outstanding SINR radar performance. The reason why this is possible is that the power centroid extracted from a SAR image range-bin is not significantly changed when the image is highly compressed as is shown in Fig. 8. This realization then led to the hypothesis that the power centroid can be extracted directly from the on-line determined sample covariance matrix ($R_{SCM}$) defined according to:

$$R_{SCM} = \frac{1}{n} \sum_{i=1}^{n} Z_i Z_i^H$$

where the set $\{Z_i=[z_{i,1},z_{i,2},...,z_{i,NM}]: i=1,...,n\}$ denotes $n$ measured samples, each complex and NM dimensional, from the range-bin in question and its immediate surroundings. Thus if the power centroid could be found directly from (32) PC-Radar will not require the use of SAR imagery, which is without doubt a major implementation advantage. Such a knowledge-unaided or KU PC-Radar scheme is described next.

**C. Knowledge Unaided PC-Radar:**

In KU PC-Radar the KA front clutter covariance matrix $(g,C_i(\theta_{AAM}))$ of (27) is replaced by a KU version $(g,C_i(\theta_{AAM}))$ defined according to:

$$KU C_i(\theta_{AAM}) = \eta \sum_{i=1}^{NM} e^{j \theta_{AAM}} e_{i}^H(\theta_{AAM})$$

where expressions (27) and (33) are the same except that $PC_{K_A}$ is replaced with its knowledge-unaided power centroid version ($PC_{K_U}$) that is defined as:
Fig. 5 Illustration of $R_{SCM}$ Moments for $PC_{KU}$ Evaluation when $N=M=3$

\[
R_{SCM} = \frac{1}{n} \sum_{i=1}^{n} Z_i Z_i^H
\]

\[
\begin{array}{cccccc}
1 & 2 & 3 & 4 & 5 = M+N-1 \\
m_1 & m_2 & m_3 & x & x & x & x & m_5 \\
x & x & x & x & x & x & x & x \\
x & x & x & x & x & x & x & x \\
x & x & x & x & x & x & x & x \\
x & x & x & x & x & x & x & x \\
x & x & x & x & x & x & x & x \\
x & x & x & x & x & x & x & x \\
x & x & x & x & x & x & x & x \\
\end{array}
\]

\[1 \leq PC_{KU} = \frac{\sum_{i=1}^{NM} e^{j\theta_{RSCM}}} {\sum_{i=1}^{NM} x_i e^{-j\theta_{RSCM}}} = \frac{NM + 1}{2} + \Delta PC(R_{SCM}) \leq NM
\] (34)

and

\[-\frac{NM - 1}{2} \leq \Delta PC(R_{SCM}) = \sum_{i=2}^{NM} \frac{\text{Im} \{m_i\}}{m_i} \leq \frac{NM - 1}{2}
\] (35)

where:
1) \{x_1,..,x_{NM}\} denotes the set of clutter powers of the $NM$ range-bin cells.
2) $(NM+1)/2$ in (34) denotes the value that $PC_{KU}$ achieves when the clutter is stationary.
3) $\Delta PC(R_{SCM})$ denotes a $R_{SCM}$ dependent non-stationary clutter correction to the power centroid of stationary clutter.
4) \{m_1,m_2,..,m_{N+M-1}\} denotes a set of first row elements of the complex $NM \times NM$ dimensional sample covariance matrix $R_{SCM}$. These $N+M-1$ elements are 2nd order statistical moments that are selected following the “column pattern” described in Fig. 5 for the $N=M=3$ case.
5) $\text{Im} \{m_i\}$ denotes the imaginary part of the $m_i$ moment.
6) \{k_i; i=2,..,N+M-1\} is a set of gains that weights the contribution of \{Imag\{m_i\}; i=2,..,N+M-1\} to the sum of imaginary moment values in (35) that results in $\Delta PC(R_{SCM})$.

In Appendix B expressions (34) and (35) are derived for the mathematically tractable case corresponding to $M=2$, $N=2$ and $\beta=1$. In particular, in (B.17) the values for $k_2$ and $k_3$ are given. Later in Section IV where radar simulation results are presented for the $M=16$, $N=16$ and $\beta=1$ case, the following simple expression will be used in determining the set of gains \{k_i; i=2,..,N+M-1\}:

\[k_i = -240 \left( \frac{-1}{2} \right)^{i}, \quad i=2,..,N+M-1
\] (36)

In Fig. 6 a block-diagram description for the $^{\text{KU}}C_i(\theta_{AM})$ derivation of (33) is shown. The block-diagram is noted to have two major components. One is a processor-encoder that derives $PC_{KU}$ according to (34)-(36). The second one is a processor-decoder that receives $PC_{KU}$ and appropriately derives values for $\eta$ and $\theta_{AM}$ to yield $^{\text{KU}}C_i(\theta_{AM})$.

From expression (33) it is once again apparent that the required on-line computations are quite taxing due to the high dimensionality of the complex multiplications. However, as noted earlier for the KA case this problem is greatly alleviated if one restricts the possible values that $PC_{KU}$ as well as $\theta_{AM}$ may have. This is what is done in the KU PC-Radar with $PC_{KU}$ quantization scheme, called here QKU PC-Radar, that is described next.

**D. Knowledge Unaided PC-Radar with $PC_{KU}$ Quantization**

In QKU PC-Radar the front clutter covariance matrix $^{\text{KU}}C_i(\theta_{AM})$ of (33) is replaced by its PC quantized version $^{\text{QKU}}C_i(\theta_{AM})$ defined according to:
where expressions (33) and (37) are the same except that $PC_{KU}$ in (33) is replaced with $Q[PC_{KU}]$ to yield (37). The quantizer leading to $Q[PC_{KU}]$ can be defined, for instance, as follows:

$$1 \leq Q[PC_{KU}] = \min_i P_{PC_{KU}} - \mid i\mid \leq NM, \quad i \in \{1 + \frac{NM - 1}{L + 1}, 1 + \frac{NM - 1}{L + 1}, \ldots, 1 + \frac{NM - 1}{L + 1}\}$$

where the value of $L$ denotes the selected number of quantization levels. These quantization levels for $PC_{KU}$ are uniformly distributed over the range-bin in this particular example.

In Fig. 7 a block-diagram description for the computation of $Q[PC_{KU}]$ according to (37) is shown. The block-diagram is noted to have two major components. One is a processor-encoder that derives $Q[PC_{KU}]$ according to (38), after first finding $PC_{KU}$ according to (34)-(36) as described earlier. On the other hand, the processor decoder is a fast lookup memory that stores off-line derived expressions for (37) corresponding to allowed quantization levels for $PC_{KU}$, $\eta$ and $\theta_{AAU}$. Alternative options available are similar to those described when the required PC quantizations for the KA case $Q[PC_{KA}]$ were considered. In the next section simulations results using real world SAR-imagery are provided.

**IV. ILLUSTRATION OF PC-RADAR SINR PERFORMANCE**

In this section, under severely taxing environmental conditions, the SINR performance of both KA and KU PC-Radar are found to approach that of an idealized DARPA KASSPER scheme, referred in our simulations as Optimum-Radar. In Optimum-Radar one uses the covariance matrix tapers approach [7] to interference plus noise covariance ($\mathcal{R}$) modeling of (7)-(8) to derive the optimum radar gain that emerges from the Wiener-Hopf equation.
Table I  Radar Simulation Values

<p>| | |</p>
<table>
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| 1. Antenna | Number of antenna elements: \(N = 16\)  
Number of CPI pulses: \(M = 16\)  
Ratio of antennal inter element spacing to wavelength: \(d/\lambda = 1/2\)  
Variance of thermal white noise (26): \(\sigma_n^2 = 1\)  
Front antenna gain constant (17): \(K_f = 56\) dBs  
Back antenna gain constant : \(K_b = -40\) dBs  
Carrier frequency (14): \(f_c = 10^9\) Hz  
Pulse repetition frequency (15): \(f_{r} = 10^3\) Hz  
Antenna array misalignment (25): \(\theta_{AAM} = 2^o\) |
| 2. Clutter | Number of Range-Bins \(N_M = 256\)  
Radar’s ratio \(\beta\) (24): \(\beta = 1\) |
| 3. Jammers | Jammers are used at the boresight angles of -60\(^o\), -30\(^o\) and 45\(^o\) with 52, 55 and 66 JNRs in dBs, respectively, inclusive of antenna gains. |
| 4. Range Walk | Fraction of remaining area after range walk (A.12): \(\rho = 0.999999\). |
| 5. Internal Clutter Motion | Shape factor, (A.14): \(b = 5.7\)  
Wind-speed, (A.16): \(\omega = 15\) mph |
| 6. Channel Mismatch: Finite-Bandwidth | Amplitude peak deviation, (A.20): \(\Delta \varepsilon = 0.001\),  
Phase peak deviation, (A.20): \(\Delta \phi = 0.1^o\) |
| 7. Channel Mismatch : Angle-Dependent | Bandwidth, (A.24): \(B = 10^8\) Hz  
Mainbeam width, (A.24): \(\Delta \theta = 28.6^o\) |
| 8. Channel Mismatch: Angle-Independent | Amplitude error (A.29): \(\Delta \varepsilon = 0\) for all i,  
Phase-error (A.29): \(\Delta \gamma_i\) fluctuates with a 5\(^o\) rms for all i |

In Table I the physical variable values that are assumed in the simulations to model major disturbance cases as well as the radar operating conditions are stated. With the exception of the front clutter covariance matrix \(C_f(\theta_{l\omega})\) (18), which is found either from the SAR imagery prior-knowledge or from simulated noisy range-bin measurements, the value of the interference covariances used in SINR evaluations is determined making use of the physical variables available from Table I. We next present our simulation results in seven subsections labeled as A thru G.

### A. Assumed SAR imagery for investigation

In our investigations the SAR image of the Mojave Airport in California displayed in Fig. 8a will be used. This 4 megabytes image is composed of 1,024 by 256 pixel elements representing 1,800 by 1,500 meters of the airport, where each pixel denotes clutter power. Sixteen consecutive rows of Fig. 8a are averaged to form the 64 x 256 image of Fig. 8b, where each row denotes a range-bin with 256 clutter cells that will be used in our simulations. The 256 clutter cells are specified in the simulation Table I as \(N_M = 256\) where \(N = M = 16\). Also in Fig. 8 one notices in Fig. 8c a compressed 512 bytes SAR image that surfaces when the 4 Mbytes SAR image of Fig. 8a is first compressed using a strip PT source-coder, and then sixteen consecutive rows of the result averaged [6]. In Figs. 8d and Fig. 8e one then views the total clutter power and clutter power-centroid, respectively, corresponding to the 64 range-bins of the uncompressed SAR image of Fig. 8b. Although the power centroids displayed in Fig. 8e were derived from the uncompressed SAR image of Fig. 8b, the power centroids derived from the compressed SAR image of Fig. 8c are not much different. This remarkable result, corresponding to a compression factor of 7,812 for the compressed SAR image, motivated the author to search for a “knowledge-unaided” scheme, i.e., one without the use of SAR imagery, where the power-centroid is derived directly from the KU sample covariance matrix (32).
B. Optimum SINR Performance (SINR*) of KA CMT-based KASSPER scheme

The optimum SINR* performance associated with a KA covariance matrix tapers based scheme is readily derived since it is assumed that the “true” clutter power of the range-bin in question is identical to that of the corresponding SAR image range-bin of Fig. 8b. In determining SINR* the physical variable values of Table I will be used to find all the prerequisite interference plus noise covariances in (7)-(8). In particular, when finding SINR* the back clutter covariance matrix \( C_b(\theta_{\lambda\lambda}) \) will be assumed negligible since the assumed back-lobes antenna gain \( K_b \) is quite small, i.e., -40 dB.

C. SINR Performance of KU sample covariance matrix (SCM) radar scheme

The SINR performance of a KU SCM-Radar scheme (\( \text{SINR}_{\text{SCM}} \)) will also be derived. It is defined according to:

\[
\text{SINR}_{\text{SCM}} = \left| w_{\text{SCM}}^H S \right|^2 / \text{tr} \left( w_{\text{SCM}}^H R_{\text{W,SCM}} w_{\text{SCM}} \right)
\]  

\[w_{\text{SCM}} = \hat{R}_{\text{SCM}}^{-1} s
\]

\[ \hat{R}_{\text{SCM}} = \frac{1}{n} \sum_{i=1}^{n} \Sigma_i Z_i^H + \sigma_{\text{diag}}^2 I \]
where: a) \( R \) is the interference plus noise covariance of (7); b) \( \hat{R}_{SCM} \) is the sample covariance matrix; c) \( \sigma_{\text{diag}}^2 \) is a diagonal loading term that addresses numerical issues linked to the \( \hat{R}_{SCM} \) inversion, a value of 10 for \( \sigma_{\text{diag}}^2 \) is used in our simulations; and d) \( \{ Z_i, i=1,..,n \} \) denotes \( n \) samples taken from the investigated range-bin and its neighbors, each complex and \( NM \) dimensional.

To derive the set of range-bin measurements \( \{ Z_i, i=1,..,n \} \) the following simulation technique is used

\[
Z_i = R_i^{-1}n_i
\]

where \( n_i \) is a zero mean, unity variance, \( NM \) dimensional complex random draw and \( R_i \) is the interference plus noise covariance (7) associated with the \( i^{th} \) range-bin and derived as described earlier for the optimum SINR* performance scheme.

D. SINR Performance of KA PC-Radar scheme

The SINR performance of the KA PC-Radar scheme (SINR\(_{KA} \)) will be investigated. It is defined according to:

\[
SINR_{KA} = \frac{|w_{KA}^H s_{\hat{w}}^1|^2}{w_{KA}^H R_{KA} s_{\hat{w}}^1}
\]

where: a) \( R \) is the interference plus noise covariance of (7); b) \( \hat{C}_{\text{\(KA\)}}^f (\theta_{\text{AMM}}) \) is the clutter covariance matrix (27) derived from KA PC-Radar; c) \( \hat{R}_{KA} \) is the estimate of \( R \) that results when \( \hat{C}_{\text{\(KA\)}}^f (\theta_{\text{AMM}}) \) replaces \( C_{\text{\(KA\)}}^f (\theta_{\text{AMM}}) \) in (7); and d) \( w_{KA} \) is the radar weighing gain of the KA PC-Radar system.

E. SINR Performance of KU PC-Radar scheme

The SINR performance of the KU PC-Radar scheme (SINR\(_{KU} \)) will also be studied. It is defined according to:

\[
SINR_{KU} = \frac{|w_{KU}^H s_{\hat{w}}^1|^2}{w_{KU}^H R_{KU} s_{\hat{w}}^1}
\]

where: a) \( R \) is the interference plus noise covariance of (7); b) \( \hat{C}_{\text{\(KU\)}}^f (\theta_{\text{AMM}}) \) is the clutter covariance matrix (33) derived from KU PC-Radar; c) \( \hat{R}_{KU} \) is the estimate of \( R \) that results when \( \hat{C}_{\text{\(KU\)}}^f (\theta_{\text{AMM}}) \) replaces \( C_{\text{\(KU\)}}^f (\theta_{\text{AMM}}) \) in (7); and d) \( w_{KU} \) is the radar weighing gain of the KU PC-Radar system.

F. SINR Performance of KU PC-Radar scheme with PC quantizations

The SINR performance of the KU PC-Radar scheme (SINR\(_{QKU} \)) with PC quantization, referred in the simulations as QKU PC-Radar, will also be studied. It is defined according to:

\[
SINR_{QKU} = \frac{|w_{QKU}^H s_{\hat{w}}^1|^2}{w_{QKU}^H R_{QKU} s_{\hat{w}}^1}
\]

where: a) \( R \) is the interference plus noise covariance of (7); b) \( \hat{C}_{\text{\(QKU\)}}^f (\theta_{\text{AMM}}) \) is the clutter covariance matrix (37) derived from QKU PC-Radar; c) \( \hat{R}_{QKU} \) is the estimate of \( R \) that results when \( \hat{C}_{\text{\(QKU\)}}^f (\theta_{\text{AMM}}) \) replaces \( C_{\text{\(QKU\)}}^f (\theta_{\text{AMM}}) \) in (7); and d) \( w_{QKU} \) is the radar weighing gain of the QKU PC-Radar system.
**G. Comparison of various schemes**

The simulation results are summarized in Figs. 9 and 10. The basic difference between the two results is in the number of quantization levels allowed for clutter centroid quantizations. Fig. 9 corresponds to eleven quantization levels and Fig. 10 to only three quantization levels. In both cases the simulations are done with three jammers at the boresight angles of -60°, -30° and 45° with corresponding JNR values of 52, 55, and 66 dBs, respectively, as noted in Table I.

Each figure has seven displays. First Figs. 9a and 10a show the SINR error in dBs as a function of range-bin where it is noted that the average SINR error over all range-bins for the SCM-Radar scheme of (39)-(42), the KA PC-Radar scheme of (43)-(45), and the QKU PC-Radar scheme of (49)-(51). For example, in Fig. 9a KA PC-Radar, QKU PC-Radar and KU SCM-Radar are noted to yield an average SINR error of 0.87 dBs, 1.32 dBs and 7.59 dBs, respectively. These results show that the KA and KU PC-Radar schemes yield more than 6 dBs improvements over SCM-Radar, while also yielding close to optimum SINR radar performance.

In Figs. 9b and 10b the total clutter plus jammer power of the KA PC-Radar, KU PC-Radar and QKU PC-Radar schemes is displayed. The two KU PC-Radar schemes are noted to yield the same total power for each range-bin. This total power, however, is also noted to deviate greatly from that of the KA PC-Radar scheme that uses in its evaluations the “true” SAR range-bin clutter plus jammer. This deviation from the true case is due to the fact that the total clutter plus jammer power derived with a KU PC-Power scheme is simply the first moment of the sample covariance matrix (32), whose value depends on noisy on-line measurements.

Next in Figs. 9c and 10c the range-bin power-centroid corresponding to KA PC-Radar, KU PC-Radar and QKU PC-Radar are displayed. From this figure it is noted that the two KU schemes yields close trajectories that roughly follow the KA PC-Radar power centroid which is once again noted to be derived from the “true” SAR rang-bin clutter plus jammer.

Finally Figs. 9d-g and 10d-g display results in dBs for the first range-bin of Fig. 8b. More specifically, in Figs. 9d and 10d the antenna gain modulated clutter is displayed for three cases. First from the actual SAR range-bin clutter covariance (18), i.e., \( \{ x_i g_i^b \ : \ i = 1, \ldots, NM \} \), second from KA PC-Radar covariance (27), i.e., \( \{ \eta g_i^{KA,\theta} \ : \ i = 1, \ldots, NM \} \) and thirdly from QKU PC-Radar (37), i.e., \( \{ \eta g_i^{QKU,\theta} \ : \ i = 1, \ldots, NM \} \). In Figs. 9e and 10e the SINR is plotted versus Doppler for the Optimum-Radar, KA PC-Radar, QKU PC-Radar and SCM-Radar cases. In Figs. 9f and 10f the adapted patterns [7] are presented for the Optimum-Radar, KA PC-Radar, QKU PC-Radar and SCM-Radar cases. Finally, in Figs. 9g and 10g the eigenvalues of the interference plus noise covariance matrix versus eigenvalues index are plotted for the Optimum-Radar, KA PC-Radar, QKU PC-Radar and SCM-Radar cases.
Fig. 9. Eleven Power Centroid Quantization Levels. (a) SINR Error. (b) Clutter Plus Jammer Power. (c) Clutter Plus Jammer Power Centroid. (d) Range-Bin #1’s Clutter. (e) Range-Bin #1’s SINR. (f) Range-Bin #1’s Adapted Pattern. (g) Range-Bin #1’s Eigenvalues
Fig. 10. Three Power Centroid Quantization Levels (a) SINR Error. (b) Clutter Plus Jammer Power. (c) Clutter Plus Jammer Power Centroid. (d) Range-Bin #1’s Clutter. (e) Range-Bin #1’s SINR. (f) Range-Bin #1’s Adapted Pattern. (g) Range-Bin #1’s Eigenvalues
The results presented in Figs. 9 and 10 are typical results for the compared schemes. The following conclusions are then drawn:

1. The SINR radar performance of both KA and KU PC-Radar emulates that of Optimum-Radar, which uses a covariance matrix tapers model for the interference plus noise covariance.

2. The SINR radar performance derived from both KA and KU PC-Radar represents a major improvement over KU SCM-Radar of more than 6 dBs for the considered illustration.

3. The KA PC-Radar scheme performs well with both uncompressed SAR imagery and “radar-blind compressed” SAR imagery because the range-bin power centroid derived from them do not vary greatly. This robustness observation led, in turn, to the discovery of “knowledge-unaided” PC-Radar.

4. The savings in on-line computational time of QKA PC-Radar over KA PC-Radar are significant because off-line computations of a small set of clutter covariance matrices permit their later fast extraction from a lookup memory.

5. The savings in storage space of KU PC-Radar over KA PC-Radar are significant because the storage of SAR imagery is unnecessary for power centroid evaluations.

6. The savings in on-line computational time of QKU PC-Radar over KU PC-Radar are significant because off-line computations of a small set of clutter covariance matrices permit their fast memory retrieves.

V. SUMMARY AND CONCLUSIONS

This paper reviewed the emergence of a fast and powerful adaptive radar method. The method surfaced naturally from the universal cybernetics duality or UC-Duality. This duality was first identified in the late 1970s in linear quadratic Gaussian control and then applied to quantized control to yield a control technique called Matched Processors. This control approach was the certainty-time dual of Matched Filters for bit detection. Moreover, it advanced a practical parallel processing approach to quantized control that did not suffer of what Bellman called “the curse of dimensionality” of his Dynamic Programming. More than two decades later the UC-Duality found another major application in knowledge aided or KA adaptive radar. More specifically, DARPA funded radar research of the early 2000s unveiled the need for a radar design theory leading to synergetic storage-space/computational-time compression solutions. The answer to this important problem was found in the UC-Duality which first revealed that information theory had a certainty-time dual that was given the name latency theory. This revelation was then followed by another which yielded latency theory’s processor-coding as the certainty-time dual of information theory’s source-coding. It was then determined that the integration of a source coder, compressing prior-knowledge such as SAR imagery, and a processor coder, compressing the computational-time associate with clutter covariance matrix evaluations, was in fact the sought after synergistic radar solution. This knowledge aided or KA solution was named PC-Radar because the task of the processor encoder was to determine the power centroid or PC of the investigated range-bin clutter for later use by the processor decoder whose task was the evaluation of the clutter covariance matrix from the measured PC. In the late 2000s a knowledge unaided or KU version of the technique was then discovered that determined the PC from an on-line derived sample covariance matrix. It was then shown using taxing environmental conditions that both KA and KU PC-Radar yielded a SINR radar performance that emulated that of a KA Optimum-Radar scheme. This was a welcome as well as extraordinary result. In the future the exposed synergistic PC-Radar ideas of this paper are expected to find extensive use in fields such as smart antennas, with applications not only found in radar but also in radio communications where non-stationary clutter interferences are the rule rather than the exception as the complexity and demands of wireless multi-media networks continuously increase. Finally, researchers working in their own challenging problems regardless of field (such as the cosmology and the biological lifespan fields investigated in Appendix C via the UC-Duality’s linger thermo theory), are encouraged to reflect on the UC-Duality counsel that says, “Synergistic physical/mathematical dualities naturally arise in efficient system designs.”
APPENDIX A
Covariance Matrix Tapers

In this appendix covariance elements of the interference plus noise covariance matrix tapers model of (7)-(8) are defined. They are:

**The Thermal White Noise Covariance:** The thermal white noise covariance \( (C_n) \) is defined according to:

\[
C_n = \sigma_n^2 I_{NM}
\]

(A.1)

where \( \sigma_n^2 \) is the average power of thermal white noise and \( I_{NM} \) is an identity matrix of dimension \( NM \) by \( NM \).

**The Jammer Covariance:** The jammer covariance matrix \( (C_j) \) is defined according to:

\[
C_j = \sum_{i=1}^{N_j} p_i g_i(\theta_i) (I_M \otimes I_{N \times N}) O (j(\theta_j) \bullet j(\theta_j)^H)
\]

(A.2)

\[
j(\theta_j) = [j_1(\theta_j), j_2(\theta_j), \ldots, j_M(\theta_j)]^T
\]

(A.3)

\[
j_k(\theta_j) = j_k(\theta_j) \quad \text{for} \ k = 1, \ldots, M
\]

(A.4)

\[
j_{i,k}(\theta_j) = [j_{i,1}(\theta_j), j_{i,2}(\theta_j), \ldots, j_{i,M}(\theta_j)]
\]

(A.5)

\[
j_{i,k}(\theta_j) = e^{j2\pi(k-1)\Delta\theta_j} \quad \text{for} \ k = 1, \ldots, N
\]

(A.6)

\[
\overline{\theta}_j = \frac{d}{\lambda} \sin(\theta_j)
\]

(A.7)

where: 1) \( N_j \) is the total number of jammers; 2) \( \theta_j \) is the boresight angle of the \( i^{th} \) jammer; 3) \( \otimes \) is the Kronecker (or tensor) product; e) \( I_M \) is an identity matrix of dimension \( M \) by \( M \); f) \( I_{N \times N} \) is a unity matrix of dimension \( N \) by \( N \); g) \( p_i \) is the \( i^{th} \) jammer power; and h) \( j(\theta_j) \) is the \( NM \times 1 \) dimensional and complex \( i^{th} \) jammer steering vector.

Finally, the first element of the \( NM \) by \( NM \) matrix \( C_j \) defines the jammer to noise ratio (JNR) which is

\[
\text{JNR} = C_j(1,1)/\sigma_n^2
\]

(A.8)

**The Range Walk Covariance:** The range walk covariance \( (C_{RW}) \) is defined according to:

\[
C_{RW} = C_{RW}^{\text{time}} \otimes C_{RW}^{\text{space}}
\]

(A.9)

\[
[C_{RW}^{\text{time}}]_{k,l} = \rho^{l-k} \]

(A.10)

\[
C_{RW}^{\text{space}} = I_{N \times N}
\]

(A.11)

\[
\rho = \Delta A / A = \Delta A / \{|\Delta R \Delta \theta| = \Delta A / \{(c / B) \Delta \theta\}
\]

(A.12)

where: a) \( c \) is the velocity of light; b) \( B \) is the bandwidth of the compressed pulse; c) \( \Delta R \) is the range-bin radial width; d) \( \Delta \theta \) is the mainbeam width; e) \( A \) is the area of coverage on the range bin associated with \( \Delta \theta \) at the beginning of the range walk; f) \( \Delta A \) is the remnants of area \( A \) after the range bin migrates during a CPI; and g) \( \rho \) is the fractional part of \( A \) that remains after the range walk.

**The Internal Clutter Motion Covariance:** The internal clutter motion covariance \( (C_{ICM}) \) is defined according to:

\[
C_{ICM} = C_{ICM}^{\text{time}} \otimes C_{ICM}^{\text{space}}
\]

(A.13)

\[
[C_{ICM}^{\text{time}}]_{k,l} = \frac{r}{r+1} + \frac{1}{r+1} \left(b \lambda^2\right) + \left(4\pi |k\cdot T_c|^2\right)^2
\]

(A.14)

\[
C_{ICM}^{\text{space}} = I_{N \times N}
\]

(A.15)

\[
10 \log_{10} r = -15.5 \log_{10} \omega - 12.1 \log_{10} f_c + 63.2
\]

(A.16)

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where: a) \( f_c \) is the carrier frequency in megahertz; b) \( \omega \) is the wind speed in miles per hour; c) \( r \) is the ratio between the dc and ac terms of the clutter Doppler power spectral density; d) \( b \) is a shape factor that has been tabulated; e) \( c \) is the speed of light; and f) \( T_r \) is the pulse repetition interval.

**The Channel Mismatch Covariance:** The channel mismatch covariance (\( C_{CM} \)) found according to:

\[
C_{CM} = C_{NB} \circ C_{FB} \circ C_{AD}
\]  

where \( C_{NB}, C_{FB} \) and \( C_{AD} \) are composite covariance matrix tapers that are defined next.

1. **The Finite Bandwidth Covariance (\( C_{FB} \)):** This is a finite (nonzero) bandwidth (FB) channel mismatch type defined according to [7]:

\[
C_{FB} = C_{time} \otimes C_{space}^{FB}
\]

\[
C_{time}^{FB} = 1_{MM}
\]

\[
[C_{space}^{FB}]_{i,k} = (1 - \Delta \varepsilon / 2)^2 \sin \left( c^2 (\Delta \phi / 2) \right) \text{ for } i \neq k
\]  

\[
[C_{space}^{FB}]_{i,i} = 1 - \Delta \varepsilon + \frac{1}{3} \Delta \varepsilon^2 \text{ for } i = 1, \ldots, N
\]  

where \( \Delta \varepsilon \) and \( \Delta \phi \) denote the peak deviations of decorrelating random amplitude and phase channel mismatch, respectively. The square term in (A.20) corrects an error in the derivation of equation (A.21) in [7].

2. **The Angle Dependent Covariance (\( C_{AD} \)):** This is an angle-dependent (AD) channel mismatch type defined according to [7]:

\[
C_{AD} = C_{time}^{AD} \otimes C_{space}^{AD}
\]

\[
C_{time}^{AD} = 1_{MM}
\]

\[
[C_{space}^{AD}]_{i,k} = \sin(\theta_k - i \frac{d}{Af_c} \sin(\Delta \theta)) \text{ for } i \neq k
\]

\[
[C_{space}^{AD}]_{i,i} = 1
\]

where \( B \) is the bandwidth of an ideal bandpass filter and \( \Delta \theta \) is a suitable measure of mainbeam width.

3. **The Angle Independent Narrowband Covariance (\( C_{NB} \)):** This is an angle-independent narrowband or NB channel mismatch type defined according to [7]:

\[
C_{NB} = [q_k H]
\]

\[
q = [q_1, q_2, \ldots, q_M]^T
\]

\[
q_k = 1 \text{ for } k = 1, \ldots, M
\]

\[
\begin{bmatrix}
\varepsilon & e^{j\pi} & e^{j2\pi} & \ldots & e^{jN\pi}
\end{bmatrix}
\]

where \( \Delta \varepsilon_1, \ldots, \Delta \varepsilon_N \) and \( \Delta \gamma_1, \ldots, \Delta \gamma_N \) denote amplitude and phase errors, respectively.

**APPENDIX B**

**Derivation of the SCM Power Centroid Expressions**

The basic idea behind the SCM power-centroid expressions of (34), (35) and (36) is explained next using the optimum \( M = N = 2 \) case depicted in Fig. B.1 for \( \beta = 1 \) as motivation. This figure shows a range-bin made of \( NM = 4 \) clutter cells which are symmetrically spaced with respect to the target that is being investigated at the boresight angle of \( \theta = \theta^o \). Thus we have that from the four boresight locations \{ \( \theta^o, \theta^o, \theta^o, \theta^o \) \} on the range-bin clutter returns are sent to the receiving two antenna elements that result in the steering vector expression
where the left element in each \( (s_i,t_i) \) pair, i.e. \( s_i \) indicates the \( i \)th antenna element and the right element \( t_j \) denotes the \( j \)th received pulse during a CPI, while the four measurement values depicted on the matrix, i.e. \( \{ z(s_i,t_j) \} \), are a function of the received modulated steering vectors as shown below

\[
z = \begin{bmatrix} z(s_1,t_1) & z(s_1,t_2) \\ z(s_2,t_1) & z(s_2,t_2) \end{bmatrix}
\]

with the four clutter steering vectors given by the expression

\[
v_i = \begin{bmatrix} v_i(s_1,t_1) & v_i(s_2,t_1) & v_i(s_1,t_2) & v_i(s_2,t_2) \end{bmatrix}^T = \begin{bmatrix} 1 & e^{\frac{2\pi j m}{\lambda} \sin \theta_i} & e^{\frac{2\pi j m}{\lambda} \sin \theta_i} & e^{\frac{4\pi j m}{\lambda} \sin \theta_i} \end{bmatrix}^T, \quad i = 1, \ldots, 4
\]

Next an expression is found for the correlation matrix \( \mathbf{E}[\mathbf{x}\mathbf{x}^H] \) under the assumption that each clutter return is uncorrelated from each other, i.e. it is assumed that \( \mathbf{E}[\mathbf{x} g_i \mathbf{x}^H g_j] = 0 \) for \( i \neq j \) and \( \mathbf{E}[\mathbf{x} g_i \mathbf{x}^H g_j] = x_{g_i} \) for \( i = j \). Thus it is found via straightforward algebraic manipulations and the symmetry condition \( \theta_2 = -22.5^\circ \) and \( \theta_1 = -67.5^\circ \) deduced from Fig. B.1 that

\[
\mathbf{E}[\mathbf{x}\mathbf{x}^H] = \begin{bmatrix} \mathbf{M}(t_1,t_1) & \mathbf{M}(t_1,t_2) \\ \mathbf{M}(t_2,t_1) & \mathbf{M}(t_2,t_2) \end{bmatrix} = \begin{bmatrix} m_1 & m_2 & m_2 & m_1 \\ m_2 & m_1 & m_2 & m_1 \\ m_2 & m_1 & m_2 & m_1 \\ m_1 & m_2 & m_1 & m_2 \end{bmatrix}
\]

where the three 2nd order correlation moments in (B.5) are found from the following three expressions

\[
m_1 = x_{g_1} + x_{g_2} + x_{g_3} + x_{g_4}
\]

\[
m_2 = x_{g_1} e^{\frac{j2\pi q}{\lambda} \sin \theta_1} + x_{g_2} e^{\frac{j2\pi q}{\lambda} \sin \theta_1} + x_{g_3} e^{\frac{j2\pi q}{\lambda} \sin \theta_1} + x_{g_4} e^{\frac{j2\pi q}{\lambda} \sin \theta_1}
\]

and

\[
m_3 = x_{g_1} e^{\frac{j4\pi q}{\lambda} \sin \theta_1} + x_{g_2} e^{\frac{j4\pi q}{\lambda} \sin \theta_1} + x_{g_3} e^{\frac{j4\pi q}{\lambda} \sin \theta_1} + x_{g_4} e^{\frac{j4\pi q}{\lambda} \sin \theta_1}
\]

Next the moment expressions (B.6)-(B.8) are used to justify the general expression (34) for \( PC_{KU} \) according to:

\[
PC_{KU} = \frac{\sum_{i=1}^{NM} \sum_{j=0}^{Nm} i x_{g_1}^i \cdot g_1^{j+\alpha}}{\sum_{i=0}^{NM} x_{g_1}^i} = \frac{NM+1}{2} + \Delta PC(R_{SCM})
\]

\[
\Delta PC(R_{SCM}) = \frac{\sum_{i=1}^{NM} \sum_{k=0}^{M} \text{Imag}[m_k]}{m_i}
\]

This is done by first noticing that the power centroid for our \( N = M = 2 \) case is given by:

\[
PC_{KU} = \frac{\sum_{i=1}^{4} i x_{g_1}^i}{\sum_{i=0}^{4} x_{g_1}^i} = \frac{x_{g_1}^1 + x_{g_2}^2 + x_{g_3}^3 + x_{g_4}^4}{x_{g_1}^0 + x_{g_2}^1 + x_{g_3}^2 + x_{g_4}^3}
\]

Then after some algebraic manipulation of (B.11) it follows that

\[
PC_{KU} = \frac{4 + 1}{2} \left( \left[ \frac{3}{2m_4} \right] x_{g_1}^4 - x_{g_2}^2 \right)
\]

where the denominator of (B.11) now appears as \( m_4 \), see (B.6), in (B.12) and the first term to the right of (B.12) is given by \( (NM+1)/2 = (4+1)/2 = 2.5 \). It is further noticed that \( (NM+1)/2 \) is the power centroid value corresponding to the boresight angle of \( 0^\circ \) for the investigated target. Secondly it is also noticed from (B.12) that the power centroid

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value of \((NM+1)/2\) arises with symmetrically distributed antenna gain weighted clutter, i.e. when \(x_3g_3=x_2g_2\) and \(x_4g_4=x_1g_1\) or when the clutter difference closest to the target, i.e. \(x_3g_3-x_2g_2\), is equal to the negative of three times the clutter difference away from the target, i.e. \(-3(x_4g_4-x_1g_1)\).

We next use expressions (B.6)-(B.8) to derive the following relationships between the real and imaginary parts of \(m_1, m_2\) and \(m_3\):

\[
\begin{bmatrix}
\text{Real}[m_1] \\
\text{Imag}[m_1]
\end{bmatrix} = \begin{bmatrix}
\cos\left(\frac{2\pi d}{\lambda} \sin \theta_1\right) & \cos\left(\frac{2\pi d}{\lambda} \sin \theta_2\right) \\
\sin\left(\frac{2\pi d}{\lambda} \sin \theta_1\right) & \sin\left(\frac{2\pi d}{\lambda} \sin \theta_2\right)
\end{bmatrix} \begin{bmatrix}
x_1g_1 + x_2g_2 \\
x_4g_4 + x_1g_1
\end{bmatrix}
\]

(B.13)

\[
\begin{bmatrix}
\text{Real}[m_1] \\
\text{Imag}[m_1]
\end{bmatrix} = \begin{bmatrix}
\sin\left(\frac{4\pi d}{\lambda} \sin \theta_1\right) & \sin\left(\frac{4\pi d}{\lambda} \sin \theta_2\right) \\
\cos\left(\frac{4\pi d}{\lambda} \sin \theta_1\right) & \cos\left(\frac{4\pi d}{\lambda} \sin \theta_2\right)
\end{bmatrix} \begin{bmatrix}
x_3g_3 - x_2g_2 \\
x_4g_4 - x_1g_1
\end{bmatrix}
\]

(B.14)

Solving the linear system of equations (B.14) for the clutter differences vector under the constraint that the matrix must be invertible, and then substituting this result in (B.12) yields the desired optimum expression for the range-bin power-centroid \(PC_{KU}\) in terms of the three correlation elements \(E[xx^H]\), i.e. \(m_1, m_2\) and \(m_3\), as follows:

\[
PC_{KU} = \frac{4+1}{2} \frac{1}{2m_i} \sum_{k=1}^{3} \begin{bmatrix}
\text{Real}[m_i] \\
\text{Imag}[m_i]
\end{bmatrix} \begin{bmatrix}
\sin\left(\frac{2\pi d}{\lambda} \sin \theta_k\right) & \sin\left(\frac{2\pi d}{\lambda} \sin \theta_k\right) \\
\cos\left(\frac{4\pi d}{\lambda} \sin \theta_k\right) & \cos\left(\frac{4\pi d}{\lambda} \sin \theta_k\right)
\end{bmatrix} \begin{bmatrix}
x_1g_1 + x_2g_2 \\
x_4g_4 + x_1g_1
\end{bmatrix}
\]

(B.15)

To get an idea of the values derived for this simple and optimum case we evaluate (B.15) for the assumed symmetrical conditions of Fig. B.1 and under the assumption that \(d/\lambda=0.5\) (also used in our simulations) to yield:

\[
PC_{KU} = \frac{5}{2} \frac{1}{2m_i} \begin{bmatrix}
\text{Imag}[m_i] \\
\text{Imag}[m_i]
\end{bmatrix} \begin{bmatrix}
\sin\left(\frac{2\pi d}{\lambda} \sin \theta_1\right) & \sin\left(\frac{2\pi d}{\lambda} \sin \theta_2\right) \\
\cos\left(\frac{4\pi d}{\lambda} \sin \theta_1\right) & \cos\left(\frac{4\pi d}{\lambda} \sin \theta_2\right)
\end{bmatrix} \begin{bmatrix}
x_1g_1 + x_2g_2 \\
x_4g_4 + x_1g_1
\end{bmatrix}
\]

(B.16)

This expression can then be rewritten as follows:

\[
PC_{KU} = \frac{5}{2} \sum_{i=2}^{3} k_i \text{Imag}[m_i] \begin{bmatrix}
\text{Imag}[m_i] \\
\text{Imag}[m_i]
\end{bmatrix} + k_2 = -2.10405 \quad \text{and} \quad k_3 = 2.17625
\]

(B.17)

where a comparison of (34)-(36) with (B.17) should help explain why the “forms” of these expressions are being used for the high dimensionality example of Section IV, inclusive of the relative location in \(E[xx^H]\) of the moments in (B.10), see (B.4) and Fig. 5 for the \(N=M=2\) and \(N=M=3\) locations, respectively.
Linger thermo theory (LTT) studies physical mediums whose mass energy ($E=Mc^2$) is regulated, i.e., it is kept constant, while interacting with its surroundings via black body radiation. $M$ denotes the medium mass, $c$ the speed of light in a vacuum and $E$ the medium energy. A brief outline of LTT will be advanced via six subsections starting with the description of four major types of system functions and ending with a novel and illuminating entropy theory for flexible-phase mediums that offers independent as well as compelling support to a previously offered LTT lifespan theory for biological systems [18], [24].

1. Four System Functions Types

Four different types of system functions characterize all mediums. Two functions are “thermal-uncertainty space” types and two are “linger-certainty time” types. While the two thermal functions pertain to the “sourcing and retention” of mass-energy that are measured with entropy metrics, the two linger functions pertain to the “processing and motion” of mass-energy that are measured with ectropy metrics.

2. The Two Thermo Entropies and Two Linger Ectropies

The two thermo entropies and two linger ectropies are defined as follows:

**The Boltzmann thermo-source entropy:** The Boltzmann thermo-source entropy ($\hat{H}$) denotes the “amount of thermal-uncertainty bits” of the system microstates according to the following “expectation uncertainty metric” (in mathematical bit units):

$$\hat{H} = \sum_{\mu_i} \Lambda_{\mu_i} \log_2(1/P[\mu_i])P[\mu_i] = \log_2 \Omega = S/k \ln 2$$  \hspace{1cm} (C.1)

where $\Lambda_{\mu_i}$ is the number of realizations of a microstate $\mu_i$ (describing a microscopic configuration of a thermodynamics system occupied with probability $P[\mu_i]$ in the course of thermal fluctuations). The expression $\log_2(1/P[\mu_i])$ denotes the “amount of thermal-uncertainty bits” associated with $\mu_i$. In addition, $\log_2(1/P[\mu_i])$ denotes the smallest possible thermal-uncertainty bits for $\mu_i$. Moreover, $\Omega$ in $\hat{H} = \log_2 \Omega$ denotes the ‘effective’ number of equally likely microstate realizations resulting in $\hat{H}$. When the microstates are equally likely it follows that $\Omega$ and $\Lambda_{\mu_i}$ would be the same. Finally, $\hat{H} = \log_2 \Omega = S/k \ln 2$ relates $\hat{H}$ to the Boltzmann “statistical” thermodynamics entropy ($S$) and constant ($k$), both in Joules/K units [15].

**The thermo-retainer entropy:** The thermo retainer entropy ($\hat{N}$) denotes the “amount of thermal-uncertainty square meters” of the system microstates according to the following “expectation uncertainty metric” (in physical SI $m^2$ units):

$$\hat{N} = \sum_{\mu_i} \Lambda_{\mu_i} 4\pi r_i^2 P[\mu_i] = 4\pi r^2$$  \hspace{1cm} (C.2)

where $\Lambda_{\mu_i}$ is the number of realizations of a microstate $\mu_i$ and $r_i$ is the expected radius of the sphere where $\mu_i$ resides when the expected shape of its volume is that of a sphere. The expression $4\pi r_i^2$ denotes the “amount of thermal-uncertainty square meters” corresponding to the surface area of a $\mu_i$ spherical volume. In addition, $4\pi r^2$ denotes the smallest possible thermal-uncertainty surface area that an arbitrarily shaped volume for $\mu_i$ could have, i.e., that of a sphere. Finally, $r$ in $4\pi r^2$ denotes an average radius for all microstate spheres.

**The linger-processor ectropy:** The linger-processor ectropy ($\hat{K}$) denotes the “amount of linger-certainty bors” of the system microstates according to the following “minimax certainty metric” (in mathematical binary operator or $\mathrm{bor}$ units):

$$\hat{K} = \max \{\log_{\mathrm{bor}} h_1, \ldots, \log_{\mathrm{bor}} h_{\Lambda_{\mu_i}} \} = \sqrt{h}$$  \hspace{1cm} (C.3)
where \( \Lambda \hat{h} \) is the number of realizations of a microstate \( \mu \), \( h \) is the number of bits for processing under \( \mu \) and \( C[\mu] \) is a “constraint” on the maximum number of inputs that a basic mathematical operator (or physical gate) can have under \( \mu \). The expression \( \log_{C[\mu]} h \) denotes the “amount of linger-certainty bors” associated with \( \mu \) where the approximation \( \log_{C[\mu]} h \approx \sqrt{h} \) holds when \( C[\mu] \) approaches the value of one and \( h \) is a very large number. In addition, \( \log_{C[\mu]} h \) denotes the smallest possible amount of linger-certainty bors of processing under \( \mu \), see [18] for the derivation of the \( \log_{C[\mu]} h \) expression and its illustration using a simple full adder example [19]. Finally under the condition \( \log_{C[\mu]} h \approx \sqrt{h} \) for all \( \mu \) the \( h \) in \( \hat{K} = \sqrt{K} \) denotes the maximum number of thermo-bit inputs linked to the microstate realization whose number of linger bors is the same as \( \hat{K} \).

**The linger-mover ectropy:** The linger-mover ectropy \( \hat{A} \) denotes the “amount of linger-certainty seconds” of the system microstates according to the following “minimax certainty metric” (in physical SI sec units):

\[
\hat{A} = \max \{\pi r / v_1, \ldots, \pi r / v_N\} = \pi r / v
\]

(C.4)

where \( \Lambda \hat{h} \) is the number of realizations of a microstate \( \mu \) and \( \bar{r} \) is the expected radius of the sphere where \( \mu \) resides when the expected shape of its volume is that of a sphere. The expression \( \pi r / v \) denotes the “amount of linger-certainty seconds” corresponding to circular rotational motion on the surface of a sphere of radius \( r \) with \( v \) denoting the rotational speed of motion in \( \mu \). In addition, \( \pi r / v \) denotes the smallest possible linger-certainty seconds for rotational motion since \( v \) is the largest possible in value [17]. Finally \( r \) and \( v \) in \( A = \pi r / v \) denote the average radius and average rotational speed for all microstate spheres, respectively.

### 3. The Universal Linger Thermo Equation

The two entropies (C.1) and (C.2) and ectropies (C.3) and (C.4) when combined produce the universal linger thermo equation (ULTE) which is a “general medium operational expression” according to [17]:

\[
\hat{H} = \log_2 \Omega = g_{Med} \left( \frac{N}{V} \right) = \frac{V}{\Delta V} = \frac{\bar{r}}{\Delta r} = \left( \frac{r}{\Delta r} \right)^2 = \left( \frac{M}{\Delta M} \right)^2 = \left( \frac{\hat{A}}{A\hat{A}} \right)^2 = \hat{K}^2
\]

(C.5)

where \( g_{Med} \) is a function that depends in the type of medium studied (e.g., a black-hole, a photon-gas or a flexible-phase medium) that relates the source/processor metrics pair \((\hat{H},\hat{K})\), with mathematical units, to dimensionless operating ratios of physical variables, inclusive of the retainer/mover metrics pair \((\hat{K},\hat{A})\). An example of a dimensionless operating ratio is \( M/\Delta M \) with \( M=E/c^2 \) denoting the mass-energy of the medium (whose value is regulated to remain constant) and \( \Delta M \) denoting an active or operating part of \( M \) called the quantum of operation (QoO) mass. The following three relationships are next highlighted for the ULTE:

1) The “mathematical units” entropy/ectropy equality

\[
\hat{H} = \hat{K}^2
\]

(C.6)

that surfaces from (C.3) when \( h \) is replaced with \( \hat{H} \).

2) The “physical units” entropy/ectropy extended relationship

\[
\hat{N} = 4\pi r^2 = 3 \frac{4\pi r^3}{3} = 3 \frac{V}{r} = 4\pi \left( \frac{GM}{v^2} \right)^2 = \frac{3\tau}{r\Pi} = \frac{4v^2\hat{A}^2}{\pi}
\]

(C.7)

that surfaces from the use of: a) equations (C.2) and (C.4); b) the equation for the escape speed \( (v_e) \) from the medium defined according to [18]:

\[
v_e^2 = 2v^2 = 2GM / r
\]

(C.8)

and derived under the assumption that the expected shape of the medium is that of a sphere of radius \( r \) whose mass-energy \( M=E/c^2 \) is modeled as a point mass residing at its center \( (v \) denotes the perpetual
rotational speed linked to the assumed point mass [18]); and c) the equation of the “life-bits pace ($\Pi$)” defined according to (in SI sec/m$^3$ units):

$$\Pi = \frac{\tau}{V} = 3\frac{\tau}{r\dot{N}}$$

(C.9)

where $\tau$ is the retention time of “thermo-bits of interest (or life-bits)” that defines a portion of the medium that leaves its expected spherical volume ($V$) via black-body radiation never to return. An example of “life-bits for a non-living system” are the thermo-bits of some compressed image such as the SAR image of Figs. 8c, residing in a medium that also contains the thermo-bits of the PT source-coder that derived the image [2]. Another example is of “life-bits for a living system” responsible for the day to day survival of an organism in a medium that also contains the thermo-bits that give the organism structure.

3) The “physical units” QoO composite expression

$$\Delta \dot{N} = 4\pi\Delta r^2 = 3 \frac{4\pi \Delta r^2}{r} = 3 \frac{\Delta V}{r} = 4\pi \left( \frac{GAM}{v^2} \right)^2 = 3\Delta \tau \frac{4v^2\Delta A^2}{\pi}$$

(C.10)

that is appropriately derived from (C.7).

4. Three ULTE examples

The ULTE is now stated for an uncharged and non-rotational black-hole, photon-gas and flexible-phase mediums, with their least “surface area” LTT expected volumes noted to be spherical in shape.

**The Black-Hole ULTE:** The black-hole (BH) ULTE is given according to [17]:

$$\dot{H}_{BH} = \frac{\dot{N}_{BH}}{\Delta N_{BH}} = \frac{V_{BH}}{\Delta V_{BH}} = \frac{\tau_{BH}}{\Delta \tau_{BH}} = \left( \frac{r_{BH}}{\Delta r_{BH}} \right)^2 = \left( \frac{M_{BH}}{\Delta M_{BH}} \right)^2 = \left( \frac{\dot{A}_{BH}}{\Delta \dot{A}_{BH}} \right)^2 = \dot{K}_{BH}^2$$

(C.11)

$$\Delta \dot{N}_{BH} = \frac{1920\ln 2}{X^c} = 7.2534 \times 10^{-70} \text{ m}^3$$

(C.12)

$$S_{BH} / k = \frac{4\pi G}{c^3} \epsilon^2 \frac{V_{BH}}{r_{BH}} = \frac{X^c}{1920} \dot{N}_{BH} = \ln 2 \dot{H}_{BH} = \ln 2 \frac{\dot{N}_{BH}}{\Delta \dot{N}_{BH}} = ...$$

(C.13)

$$k T_{BH} = \left( \frac{c(S_{BH} / k)}{\epsilon} \right)^{-1} = \frac{E_{BH}}{2S_{BH}} = \frac{c^4 h}{8\pi G E_{BH}}$$

(C.14)

$$\chi = \frac{T_{BH}}{V_{BH}} = 480 \frac{c^2}{hG} = 6.1203 \times 10^{63} \text{ s/m}^3$$

(C.15)

$$\Delta \tau_{BH} = 640 \ln 2 \frac{r_{BH}}{c}$$

(C.16)

$$\Delta M_{BH} = \ln 2 \chi / 4\pi G = 5.1152 \times 10^{-9} \text{ kg}$$

(C.17)

$$\phi E_{\text{L-Bh}} = \phi M_{\text{L-Bh}} c^2 \left[ 1 - \sqrt{1 - \Delta M_{BH}^2 / M_{BH}^2} \right] M_{BH} c^2$$

(C.18)

where all the variables were either implicitly or explicitly defined earlier in (C.1)-(C.10) except for: a) $T_{BH}$ denoting the temperature of the black-hole; b) $G$ denoting the gravitational constant; c) $\dot{h}$ denoting the reduced Planck constant; d) $\chi$ denoting the “pace of dark in a black hole” ($\chi$ is the retention dual of motion’s “speed of light in a vacuum $c$”, noted from (C.15) to be the ratio of the duration of life-bits in the black-hole ($\tau_{BH}$) over its initial volume $V_{BH}$—with all the thermo-bits in this volume assumed to be life-bits, i.e., thermo-bits of interest, whose radiation by the black-hole decreases its mass-energy until it completely evaporates); and e) $\phi E_{\text{L-Bh}}$ denotes the quantum of radiation (QoR) energy of the “single” life-bit emitted during the black-hole QoO lifespan $\Delta \tau_{BH}$ [17].

**The Photon-Gas ULTE:** The photon-gas ULTE is defined according to [17]:

$$\dot{H}_{PG} = \frac{\dot{N}_{PG}}{\Delta N_{PG}} = \frac{V_{PG}}{\Delta V_{PG}} = \frac{\tau_{PG}}{\Delta \tau_{PG}} = \left( \frac{r_{PG}}{\Delta r_{PG}} \right)^2 = \left( \frac{M_{PG}}{\Delta M_{PG}} \right)^2 = \left( \frac{\dot{A}_{PG}}{\Delta \dot{A}_{PG}} \right)^2 = \dot{K}_{PG}^2$$

(C.19)

$$\Delta \dot{N}_{PG} = (\ln 2) 135 \frac{c^3 h^3}{4 \pi^2 (kT)^3 r_{PG}}$$

(C.20)

$$S_{PG} / k = \frac{16\pi^2 (kT)^3 G^2 E_{PG}^3}{135 c^2 h^3 r_{PG}^3} = \frac{4\pi^2 (kT)^3 r_{PG}}{135 c^2 h^3} \dot{N}_{PG} = \ln 2 \dot{H}_{PG} = \ln 2 \frac{\dot{N}_{PG}}{\Delta \dot{N}_{PG}} = ...$$

(C.21)
The Flexible-Phase ULTE: The novel flexible-phase ULTE is defined according to:

\[
\hat{H} = J \log_2 \left( \frac{N}{\Delta N} \right) = V \frac{r}{\Delta r} = \left( \frac{M}{\Delta M} \right)^2 = \left( \frac{\Delta}{\Delta A} \right)^2 = \hat{k}^2
\]

(C.23)

\[
\Delta N = J^\beta \left( \frac{\beta c_v J k T}{E} \right)^{\beta E_v} \left( m k T \right)^{2/3} \frac{1}{V} \hat{N} = \frac{3 J^\beta}{g r} \left( \frac{m k T}{2 \pi h^2} \right)^{2/3} \left( \frac{m k T}{2 \pi h^2} \right)^{1/2}
\]

(C.24)

\[
S / k = \ln \Omega = J \ln \left( \frac{g}{J^\beta \left( \frac{E}{\beta c_v J k T} \right)^{e_v} \left( m k T \right)^{2/3} \left( V / \Delta N \right) \left( m k T \right)^{2/3}} \right) = \ln \frac{\hat{N}}{\hat{k}^2} = \ln \frac{v}{\Delta v} = \ln \frac{V}{\omega} = \ln \frac{E}{\hat{k}^2}
\]

(C.26)

\[
k T = \left( \frac{\partial (S / k)}{\partial E} \right)^{-1} = k \frac{1}{\beta c_v} \ln \left( \frac{g}{J^\beta \left( \frac{E}{\beta c_v J k T} \right)^{e_v} \left( m k T \right)^{2/3} \left( V / \Delta N \right) \left( m k T \right)^{2/3}} \right) = k J \frac{\Delta E^0}{E_{\hat{k}^2}} = k J \ln \frac{\tau}{\Delta \tau} = ... = k \ln 2 \hat{H}
\]

(C.27)

\[
S = J \frac{\Delta S}{\Delta J} = J \frac{Q / T}{\Delta M / \Delta m} = k J \frac{Q}{\Delta M / \Delta m} = k J \frac{\Delta E^0}{E_{\hat{k}^2}} = k J \frac{\Delta E^0}{E_{\hat{k}^2}} = k J \ln \frac{\tau}{\Delta \tau} = ... = k \ln 2 \hat{H}
\]

(C.28)

where: 1) \( g \) is dimensionless and denotes the degeneracy of the ground energy state of the medium, e.g., for a water medium it has a value of one; 2) \( T \) denotes the medium temperature, e.g., \( T=310 \) K for liquid water (this special medium will be used here to model that of a 70 kg individual since more than 98% of our molecules are those of water which together contribute to more than 65% of our total mass); 3) \( m \) denotes the mass of a “massive particle” such as an atom or molecule, e.g., \( m=3 \times 10^{-26} \) kg for a \( \text{H}_2\text{O} \) molecule, whose De Broglie wavelength is \( \lambda=\sqrt{2 \pi h^2 / m k T} \) that forms part of the entropy expression (C.25); 4) \( c_f \) is the heat capacity of a medium with constant volume, e.g., \( c_f=3 \) for liquid water at 310 K; 5) \( \beta \) is a DoF coupling constant [21] that reflects the non-equilibrium thermal conditions of the medium, while \( \alpha \) is a constant assumed equal to 0.8132 for our running example (e.g., when \( \beta=0.83 \) and \( c_f=3 \) we derive \( \tau=102 \) years and \( c_i=2.49 \), with this \( c_i \) value denoting the heat capacity of the individual which is a reasonable result); 6) \( \beta c_v J k T \) denotes the energy of a theoretical “thermal-energy particle”, e.g., \( \beta c_v J k T=1.0657 \times 10^{-20} \) Joules for our running example (as a means of comparison the energy of an electron is \( 8.187 \times 10^{-19} \) Joules); 7) \( E=Mc^2 \) is the “internal mass-energy” of the medium, e.g., for 70 kg of water, i.e., \( M=70 \) kg, one derives \( E=6.28 \times 10^{18} \) Joules (as a means of comparison the internal energy (U) for an ideal gas model, which unlike the LTT flexible-phase model does not include the medium mass-energy, is given by \( U=c_v J k T M / m = 10^{18} \) Joules when \( T=1045 \) K and the \( c_v \) and \( M \) and \( m \) values are those of our running example); 8) \( J=E/\beta c_v J k T \) is the number of thermal-energy particles in \( E \), e.g., \( J=5.9032 \times 10^{20} \) and \( J_{\text{QoO}}=1.48 \times 10^{16} \) for our running example, where it is noted that the negative logarithmic term \( J\ln(1/J_{\text{QoO}}) = J \ln(J_{\text{QoO}}) \) appearing in (C.25) denotes a \( c_f \) based correction of the dimensionless entropy \( S/k \) due to the indistinguishability of the J thermal-energy particles; 9) \( Q \) is the QoO heat energy entering the medium during \( \Delta \tau \), e.g., \( Q=7.5825 \times 10^{18} \) Joules for a human consuming 1,814 kcal per day where \( \Delta \tau=1 \) day and the conversion factor of \( \mu=4.18 \) Joules/cal is used; 10) \( \Delta S=Q/T \) is the Clausius entropy [15] contributed to the medium at temperature \( T \) by \( Q \) during \( \Delta \tau \), e.g., \( \Delta S=2.446 \times 10^{10} \) Joules/K for our example; 11) \( \Delta M = Q / \Theta \mu \) is the mass equivalent for the energy \( Q \) that is expressed as the ratio of \( Q \) to the product of \( \Theta \) and \( \mu \) with \( \Theta=5,000 \) kcal/kg and \( \mu=4.18 \) Joules/cal, e.g., \( \Delta M=0.3628 \) kg for our example; 12) \( \Delta m=k T \ln (\tau/\Delta \tau)/\Theta \mu \) denotes a fraction of the massive particle \( m \) (or QoO \( m \)) that is expressed as the ratio of the lifespan-weighted thermal-energy term \( k T \ln (\tau/\Delta \tau) \) to product of \( \Theta \) and \( \mu \), e.g., \( \Delta m=2.1538 \times 10^{27} \) kg when \( \Delta \tau=1 \) year=1/365 year and the lifespan of \( \tau \) of the life-bits in the medium is of 102 years (as a means of comparison the mass of a hydrogen atom \( m_{\text{H}} \) is \( 1.6667 \times 10^{27} \) kg); 13) \( Q^0_{\text{QoO}}=Q \) is the QoR energy that leaves the medium during \( \Delta \tau \) and is the same as the operating heat energy \( Q \) that enter it (this operation is a control or compensating action from the surroundings of the medium that maintains the medium mass-energy \( E=Mc^2 \)).
constant with the passing of time); 14) \( \Delta J = \Delta M / \Delta m = Q / kT \ln(\tau / \Delta \tau) \) denotes the fraction of the total number of thermal-energy particles \( J \) of the medium which equals the ratio of \( \Delta M \) to \( \Delta m \) or equivalently the ratio of \( Q \) to \( kT \ln(\tau / \Delta \tau) \) which is 1.48 x 10^{26}; 15) \( \Delta E_{\Delta \tau}^{LB} = \Delta kT = \Delta E_{\Delta \tau}^{LB} / \ln(\tau / \Delta \tau) \) denotes a ‘life-bits (LBs) energy’ fraction of the QoR radiation energy \( \Delta E_{\Delta \tau}^{LB} \); and 16) \( N_{LB}^{\Delta \tau} = \Delta E_{\Delta \tau}^{LB} / \Delta E_{\Delta \tau}^{LB-1} \) denotes a “hypothetical black-hole based” number of life-bits that leave the medium during \( \Delta \tau \), and is defined as the “QoR energy ratio” of the life-bits energy \( \Delta E_{\Delta \tau}^{LB} \) that leaves the medium during \( \Delta \tau \) over the QoR energy \( \Delta E_{\Delta \tau}^{LB-1} \) of the single life-bit that leaves a black-hole over its QoO lifespan \( \Delta \tau_{BH} = 640 \ln(2r_{BH} / c) \) with the mass of the black-hole being the same as that of the flexible-phase medium, e.g., \( N_{LB}^{\Delta \tau} = 64 \times 10^6 \) bits = 8 Mbytes where from (C.18) one finds \( \Delta E_{\Delta \tau}^{LB-1} = 0.0112 \) Joules for our running example. It is also of interest to note that for our running example the number of life-bits \( N_{LB}^{\Delta \tau} \) (or QoR energy ratio) is close to 90% of the “QoO mass ratio” \( \Delta M_{BH} / \Delta m = 71 \times 10^6 \) where the QoO mass \( \Delta M \) is 0.3628 kg and \( \Delta M_{BH} \) is the black-hole’s fixed QoO mass of 5.1152 x 10^{-9} kg, see (C.17).

5. The LTT Flexible-Phase Entropy versus the Non-LTT Ideal-Gas Entropy Forms

Since the LTT Flexible-Phase Entropy equation of (C.25) is similar in form to that of a non-LTT Ideal-Gas Entropy expression [20] the internal structures of these two expressions are contrasted next for further insights.

First the LTT flexible-phase entropy is noted to be described by the following four expressions:

- The LTT flexible-phase Boltzmann entropy \( (S) \) from (C.25) according to:
  \[
  S = k \ln \Omega = kJ \ln \left( \frac{E \beta c \sqrt{V}}{mkT} \right) \sqrt{\frac{mkT}{2\pi e^2}} \ln \left( \frac{mkTc^3}{2\pi e} \right) \quad (C.29)
  \]

- The LTT flexible-phase thermal-energy \( (kT) \) according to:
  \[
  kT = \frac{\partial (S / k)}{\partial E} = E / \beta c \quad (C.30)
  \]

- The LTT flexible-phase law according to:
  \[
  VP = kT = E / \beta c \quad (C.31)
  \]

- The LTT flexible-phase number of particles according to:
  \[
  J = E / \beta c \quad (C.32)
  \]

where all of the variables for (C.29)-(C.32) were earlier defined for (C.25) except for \( P \) in (C.31) which is the LTT pressure experienced by the flexible phase medium.

Secondly the non-LTT ideal-gas (IG) entropy is noted to be described by the following four expressions:

- The non-LTT ideal-gas Boltzmann entropy \( (S_{IG}) \) according to:
  \[
  S_{IG} = k \ln \Omega_{IG} = k\lambda_{IG} \ln \left( \frac{E}{U_{IG}} \left( \frac{U}{c_v \lambda_{IG} kT_{IG}} \right)^{3/2} \frac{mkT_{IG}}{2\pi e^2} \ln \left( \frac{mkT_{IG}c}{2\pi e} \right) \right) \quad (C.33)
  \]

- The non-LTT ideal-gas thermal-energy \( (kT_{IG}) \) according to:
  \[
  kT_{IG} = \left( \frac{\partial (S_{IG} / k)}{\partial U} \right)^{-1} = U / c_v \lambda_{IG} 
  \]

- The non-LTT ideal-gas law according to:
  \[
  V_{IG} P_{IG} = \frac{J_{IG} kT_{IG}}{c_v} = U / c_v 
  \]

- The non-LTT ideal-gas number of particles according to:
  \[
  J_{IG} = \frac{U}{c_v \lambda_{IG} kT_{IG}} = M / m 
  \]

where a comparison of (C.29)-(C.32) and (C.33)-(C.36) reveals five basic internal structural differences between these expressions. They are:
1) The LTT internal energy \( E \) in (C.29) is the total energy of the FP medium, thus it includes all gravitational/non-gravitational interactions of its particles, while the non-LTT internal energy \( U \) in (C.33) is the IG medium energy that only includes the kinetic-energy/potential-energy of its particles.

2) The LTT number of particles \( J \) in (C.29) refers to “thermal-energy particles” of the FP medium, while the non-LTT number of particles \( J_{IG} \) in (C.33) refers to “massive particles” of the IG medium.

3) The LTT pressure \( P \) in (C.31) refers to the pressure exerted on the FP medium volume by all the thermal-energy particles of the internal energy \( E \), while the non-LTT pressure \( P_{IG} \) in (C.35) refers to the pressure exerted on the IG medium by all the massive particles making up the internal energy \( U \).

4) The LTT DoF coupling constant \[ \beta \] in (C.29) alters the heat capacity or DoF of a medium assumed in thermal equilibrium to reflect actual non-equilibrium thermal conditions, while the non-LTT ideal-gas entropy model of (C.33) does not have such coupling constant for non-equilibrium thermal conditions.

5) The LTT number of microstates \( \Omega \) in (C.29) is exponentially related to the number of thermal-energy particles \( J \), while the non-LTT number of microstates \( \Omega_{IG} \) in (C.33) is exponentially related to the number of massive particles \( J_{IG} \). Thus the LTT Boltzmann entropy \( S \) is expected under most conditions to be significantly larger than the non-LTT Boltzmann entropy \( S_{IG} \) since \( J \gg J_{IG} \).

6. Application of the Flexible-Phase ULTE to Biological Lifespan Studies

The flexible-phase ULTE of (C.23)-(C.28) is next applied to the modeling of a biological medium which due to its non-gas state requires a medium model reflecting both gravitational and non-gravitational atomic/molecular interactions. Before the ‘total’ internal energy of the medium \( E \) was first used in [22] to investigate the entropy of flexible-phase mediums, the non-LTT Boltzmann entropy for an ideal gas medium was used by the author as a simple entropy model for a biological medium [23]. Clearly the predictions of this simple ideal gas method were rather limited in their scope since they did not reflect atomic or molecular interactions of both gravitational and non-gravitational origin. Fortunately, the nascent LTT flexible-phase ULTE method sensibly improves on this state of affairs as it is now shown. First it is noted that (C.25) and (C.26) as well as the mass density equation for a water medium given by \( V=M/1000=E/1000c^2 \), lead to the following theoretical adult lifespan \( \tau \) expression and specific result of \( \tau = 102 \) yrs for a 70 kg individual that consumes 1,841 kcal/day of food energy:

\[
\tau = \Delta \tau \frac{g}{J} \left( \frac{mkT}{2\pi \hbar^2} \right)^{3/2} \quad V = \Delta \tau g \left( \frac{\beta \pi kT}{Mc^2} \right)^{3/2} \frac{M}{1000} \approx 102 \text{ yrs} \tag{C.37}
\]

where \( \alpha=0.8132, J=Mc^2/\beta \pi kT, V=M/1000, \Delta \tau = 1/365 \) year, \( g=1, m=3 \times 10^{-26} \) kg, \( T=310 \) K, \( M=70 \) kg, \( c_v=3 \) and \( \beta=0.83 \). The \( \tau = 102 \) years of (C.37) is then noted to match the following result (US patent allowed [24]):

\[
\tau = \Delta \tau (M/\Delta M) \approx 102 \text{ yrs} \quad \tag{C.38}
\]

that surfaces from the ULTE (C.5) where \( M=70 \) kg and \( \Delta M=0.3628 \) kg (for a 1,841 kcal/day diet) have been used.

The two different approaches to the evaluation of \( \tau \) expressed by (C.37) and (C.38) have resulted in identical results when reasonable assumptions are made for the daily consumption of energy by a 70 kg individual. This result suggests that both approaches advance sensible as well as viable tools for the study of biological lifespan. While the mass-lifespan equation (C.38) represents a ‘physical-macro nutritional consumption rate’ approach to adult human lifespan studies [24], the entropy-lifespan equation (C.37) represents a ‘mathematical-micro DoF’ approach. It is further noticed that if one assumes that the childhood lifespan of the 70 kg individual is of 18 years, his/her expected total lifespan would be 120 years (this number is close to the maximum recorded lifespan for a human which exceeds 122 years). Moreover, lower adult lifespans would be found if the daily consumption of food is greater than 0.3628 kg which, in turn, produces a DoF coupling decrease reflected by a \( \beta \) larger than 0.83. For instance, when \( \Delta M=0.5654 \) kg for a 2,827 kcal/day diet and \( \beta=0.8424 \) one derives a \( \tau \) of 42 years from both (C.37) and (C.38). Notice that this decrease in adult lifespan from 102 to 42 years is substantial but expected since the 70 kg individual has significantly increased the amount of kilocalories of energy consumption per day “while still maintaining the same mass of 70 kg.” Clearly this implies an increased metabolic stress or “system usage” which is thought to lead to the individual aging at a faster rate [25], [24]. Finally, it is noted that \( \Delta M \) can be readily expressed as a non-linear function of \( \beta \) after expressions (C.37) and (C.38) are equated to yield:

\[
\Delta M = \sqrt{\left(2\pi \hbar^2 / mkT\right)^{3/2} \left(\frac{Mc^2}{\beta \pi kT}\right)^{3/2} (1000M / g)} \quad \tag{C.39}
\]
DEDICATION
This invited paper I dedicate to the memory of Norbert Wiener (1894 – 1964), originator of Cybernetics.

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REFERENCES

Proc. of SPIE Vol. 9120 912007-29