Latency-Information Theory and Applications, Part III: On the Discovery of the Space Dual of the Laws of Motion in Physics

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ABSTRACT

In this third of a multi-paper series the *discovery* of a space dual for the *laws of motion* is reported and named the *laws of retention*. This space-time duality in physics is found to inherently surface from a latency-information theory (*LIT*) that is treated in the first two papers of this multi-paper series. A motion-coder and a retention-coder are fundamental elements of a LIT's recognition-communication system. While a LIT's motion-coder addresses *motion-time issues* of knowledge motion, a LIT's retention-coder addresses *retention-space issues* of knowledge retention. For the design of a motion-coder, such as a modulation-antenna system, the laws of motion in physics are used while for the design of a retention-coder, such as a write/read memory, the newly advanced laws of retention can be used. Furthermore, while the laws of motion reflect a *configuration of space certainty*, the laws of retention reflect a *passing of time uncertainty.* Since the retention duals of motion concepts are too many to cover in a single publication, the discussion will be centered on the retention duals for Newton's Principia and the gravitational law, Coulomb's electrical law, Maxwell's equations, Einstein's relativity theory, quantum mechanics, and the uncertainty principle. Furthermore the retention duals will be illustrated with an uncharged and non-rotating black hole (*UNBH*). A UNBH is the retention dual of a vacuum since the UNBH and vacuum offer, from a theoretical perspective, the least resistance to knowledge retention and motion, respectively. Using this space-time duality insight it will be shown that the *speed of light* in a vacuum of c_M=2.9979 x 10^8 *meters/sec* has a retention dual, herein called the *pace of dark* in a UNBH of c_R=6.1123 x 10^{63} *secs/m*^{3} where 'pace' refers to the expected retention-time per retention-space for the 'dark' knowledge residing in a black hole.

Keywords: space-time dual, time dual, latency, latency theory, information, latency-information theory, sourced-space, motion-time, processing-time, retention-space, bits, bors, processor ectropy, source entropy, sensor consciousness, channel capacity, knowledge aided, intelligent system, DARPA, KASSPER, laws of motion, laws of retention, motion coder, retention coder, speed of light, pace of dark, retention, motion, motion ectropy, retention entropy, physics, biology

1. INTRODUCTION

The laws of motion originated with Newton's 1687 Principia [1] which laid out the mathematical principles of time, force, and motion that have served over more than three centuries as the essential catalyst for significant innovations. A fundamental application of these laws is in the design of a communication system [2]. This is the case since the objective of any communication system is to achieve knowledge motion with the least possible use of motion-time while subjected to design constraints. For instance, the designer of an *electrical* communication system will use Maxwell's equations and spectral analysis tools to design a modulation-antenna subsystem, or motion-coder, for knowledge motion. The space-dislocated knowledge for a 'general' communication system can be anything, e.g., the position and/or velocity of an object, the spin state of a photon, the charge of a fundamental particle, etc. On the other hand, the motion-time or lifetime penalty associated with knowledge motion cannot be avoided since there is an upper limit on the speed of motion that is given by the *speed of light in a vacuum* of approximately 2.9979 x 10^8 *meters/sec*. Nevertheless, when addressing motion problems one selects according to the application at hand the appropriate laws of motion in physics

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to use. Some of these laws are Newton's Principia and the gravitational law of classical mechanics, Coulomb's electrical law, Maxwell's electromagnetism equations, Einstein's special and general relativity, quantum mechanics, the uncertainty principle, relativistic quantum mechanics, etc. Furthermore, all of these laws reflect a configuration of space certainty associated with knowledge motion, even if one is dealing with wave motions or as is the case in quantum mechanics a probabilistic interpretation is advanced for the location and/or velocity of small objects and/or fundamental particles.

Unfortunately, however, the aforementioned laws of motion do not address another fundamental problem in physics that prominently surfaces from the time dual of a communication system, i.e., a recognition system [3]-[4]. This physics problem is that a recognition system requires a retention-coder for knowledge retention. A retention-coder is a write/read device for the retention of prior-knowledge. The recognition system's retention-coder is the space dual of a communication system's motion-coder. While the motion-coder design is concerned with motion-time (or lifetime) penalty issues of knowledge space motion, the retention-coder design is concerned with retention-space (or lifespace) penalty issues of knowledge time retention. Also while motion problems are governed by a *configuration of space certainty*, retention problems are governed by a *passing of time uncertainty*. Soon after I discovered latency theory's sensor coding as the time dual of information theory's channel coding in the summer of 2006 [5], I realized that for the laws of motion in physics, which addresses *motion-time penalty* issues, there must be a 'laws of retention in physics' dual which addresses *retention-space penalty* issues. Since this time I have researched this problem with the first retention-motion (or space-time) duality in physics results reported here. To illustrate this space-time duality an uncharged and non-rotating black hole (*UNBH*) will be used [6]. A UNBH is the retention dual of a vacuum since the UNBH and vacuum offer, from a theoretical perspective, the least resistance to knowledge retention and motion, respectively. In other words, while knowledge suffers the least lifetime penalty when moved thru a vacuum (e.g., a laser signal pays a lifetime penalty of approximately 15 msec when moved from New York to California via a fiber optics channel), knowledge suffers the least lifespace penalty when retained in a black hole (e.g., one kilogram of mass pays a lifespace penalty of approximately 1.3839 x 10^{-80} cubic meters when retained for 1,846 years in a UNBH [7]). Using this space-time duality for physics insight it is shown in Appendix A that the speed of light c_M in motion-time/motion-space units has a retention dual, herein called the *pace of dark* c_R, in expected retention-time per retention-space units given by

$$c_R = \frac{480 c_M^2}{\hbar_M G_M} = 6.1123 \text{ x } 10^{63} \text{ secs/m}^3 \quad (1.1)$$

where $\hbar_M$ =1.0546 x 10^{-34} $kg.m^2/sec$ is Plank's reduced constant of quantum mechanics and G_M = 6.693 x 10^{-11} Nm^2/kg^2 is the gravitational constant. Pace in 'pace of dark' refers to the expected retention-time per retention-space for the 'dark' knowledge residing in a black hole. Thus just like the value of c_M tell us that knowledge cannot be moved at a rate faster than 2.9979 x 10^8 meters per second, the value of c_R tell us that knowledge cannot be retained at an expectation rate faster than 6.1123 x 10^{63} seconds per cubic meter of space.

The paper is organized as follows. It begins with the discussion of the recognition/communication system in latency-information theory (*LIT*) which integrates in one picture four subsystems. The first two are the standard channel and source integrated (CSI) and sensor and processor integrated (SPI) coders of LIT [3]-[4] and the next two are a motion-coder and a retention-coder that naturally arise from LIT. In the next section in the same spirit as done with CSI and SPI coders, novel motion-time and retention-space bounds are defined for guidance in the design of motion-coders and retention-coders. Subsequently, starting first with Newton's Principia the corresponding retention duals are found for the laws of motions. The paper ends with a conclusions section.

2. LATENCY-INFORMATION THEORY (LIT)

Latency-information theory provides performance bounds that guide the design of the general system displayed in Fig. 1. This general system consists of a communication system embedded in a recognition system [3]. In [3] a detailed description is given of the communication system and recognition system subsystems appearing in this figure. Nevertheless a succinct summary of these subsystems is advanced next. While a communication system is composed of a channel, source-coder, channel-coder and motion-coder, the time dual of a communication system, i.e., a recognition system, is composed of a sensor, processor-coder, sensor-coder and retention-coder. A channel is the medium through which knowledge must be space dislocated. A source-coder (encoder/decoder) replaces an inefficient signal-source with one yielding the *smallest sourced-space* (in *b*inary dig*its* (*bits*)) *penalty possible.* A channel-coder identifies the necessary overhead-knowledge for a more accurate transmission of the sourced-space. A motion-coder is a transmitter/receiver device that moves knowledge from one location in space to another with the *smallest motion-time penalty possible*. A recognition system, as discovered in [3], uses prior-knowledge about a signal-processor's input to enable the sensing of its output by a processing-time limited sensor when the fastest possible signal-processor replacement cannot achieve this task. A sensor is the time dual of a channel and is the reason for the required time-dislocation of the processing-time to an earlier time via the use of prior-knowledge. The processor-coder is the time dual of the source-coder and replaces the signal-processor with one yielding the *smallest processing-time* (in *b*inary operat*ors* (*bors*)) *penalty possible*. The sensor coder is the time dual of the channel-coder and its purpose is to identify the necessary prior-knowledge for an earlier beginning of the processing-time. A retention-coder is a write/read device that retains knowledge from one time instant to another with the *smallest retention-space penalty possible*. Information-theory uses the sourced-space performance bound penalties of source-entropy $\boldsymbol{H}$ and channel-capacity $\boldsymbol{C}$ to guide the design of channel and source integrated (*CSI*) coders. On the other hand, latency-theory uses the processing-time performance bound penalties of processor-ectropy $\boldsymbol{K}$ and sensor-consciousness $\boldsymbol{F}$ to guide the design of sensor and processor integrated (*SPI*) coders. While the bounds $\boldsymbol{H}$ and $\boldsymbol{C}$ are governed by the uncertainty associated with the passing of time, the bounds $\boldsymbol{K}$ and $\boldsymbol{F}$ are governed by the certainty associated with the configuration of space. Finally, the laws in physics are used in the design of motion-coders and retention-coders. While the laws of motion are used for the design of motion-coders, in this paper a newly discovered space dual for the laws of motion, herein named the laws of retention, will be advanced for the design of retention-coders. In section 4 we will derive the laws of retention. However, first in the next section we will define performance bounds for use in the design of motion-coders and retention-coders.

3. MOTION-CODER AND RETENTION-CODER PERFORMANCE BOUNDS

The motion-coder performance bound, i.e., the motion-ectropy $\boldsymbol{A}$, and the retention-coder performance bound, i.e., the retention-entropy $\boldsymbol{N}$, defined here are similar in structure to those defined for a processor-coder, i.e., the processor-ectropy $\boldsymbol{K}$, and a source-coder, i.e., the source-entropy $\boldsymbol{H}$, respectively, in [3]-[4].

3.1 Motion-Ectropy $\boldsymbol{A}$: The Motion-coder Performance Bound

In the case of a motion-coder its motion-ectropy $\boldsymbol{A}$ in motion-time (or lifetime) penalty units is governed by a configuration of space certainty similar in nature to that which governs the processor-ectropy $\boldsymbol{K}$ in bors of SPI coders. Thus the motion-ectropy $\boldsymbol{A}$ has a similar minimax mathematical structure [3]-[4] in its definition, i.e.,

$$\boldsymbol{A} = \max(L_M(z_1),..,L_M(z_M)) \quad \text{in } \textit{seconds} \text{ per } \mathbf{z} = \left[z_1,\ldots,z_M\right] \tag{3.1}$$

where $\mathbf{z}$ is the motion-decoder vector output with M elements $\{z_i\}$ and $L_M[z_i]$ is the *motion-latency* of z_i which is defined as the minimum motion-time that is needed to obtain z_i after the original motion system is redesigned subject to implementation motion constraints $\{C_M [z_i]\}$ [3]. For instance, the original motion system can be an automobile that is used to transport a family of five (M=5) from New York to California and the redesigned motion system is implemented subject to the implementation motion constraint that one or more commercial airline planes may be used to transport the family. Thus

$$L_M(z_i) = g(C_M [z_i]) \quad \text{in } \textit{second} \text{ per } z_i \tag{3.2}$$

with $g(C_M[z_i])$ indicating that $L_M (z_i)$ is a function of $C_M[z_i]$. For instance, in the case of our running example the value of $L_M(z_i)$ can be quite different for each member of the family, since it is possible for all the members of the family to travel to California using different planes. In all the computations it will also be assumed that the constraint $C_M[z_i]$ is governed by a *configuration of space certainty* as is the case in latency theory [3]. For instance, for our running example we will assume that the automobile or plane(s) will always leave and arrive on time. The design of a motion-coder is then approached using $\boldsymbol{A}$ as a motion-time (or lifetime penalty) performance bound for the desired knowledge space-dislocation through motion-space. For instance, for our running example the value of $\boldsymbol{A}$ for our family of five can be of six hours of travel time from New York to California. Similarly as for a processor-coder [3], for which a processor-coder rate R_{PC} is defined, a *motion-coder rate* R_{MC} can now be defined which leads to the definition of either a *lossless or lossy* motion-coder. A motion-coder will be lossless when R_{MC} is achievable, i.e.,

$$\boldsymbol{A} \le R_{MC} \le R_M \tag{3.3}$$

where R_M is the motion rate of the original motion system. For instance, for our running example R_M may be given by the 48 hours that the family of five will take to travel by automobile from New York to California. A motion-coder will be ideal when $R_{MC}=\boldsymbol{A}$ and is equivalent to the original motion system when $R_{MC}=R_M$. On the other hand, a lossy motion-coder is one that has an R_{MC} that is not achievable, i.e.,

$$0 \le R_{MC} < \boldsymbol{A} \tag{3.4}$$

but is faster and simpler than a lossless motion-coder. For instance, for our running example we will have a lossy motion-coder when the movement from New York to California of the five member family is only implemented for the member of the family exhibiting the smallest motion-latency value, say of five hours, which is less than the six hours given by $\boldsymbol{A}$. We will also achieve a significant implementation simplicity and airfare savings.

3.2 Retention-Entropy $\boldsymbol{N}$: The Retention-coder Performance Bound

In the case of a retention-coder its retention-entropy $\boldsymbol{N}$ in retention-space (or lifespace) penalty per stored mass and/or energy is governed by a passing of time uncertainty similar in nature to that governing Shannon's source-entropy $\boldsymbol{H}$ in bits of CSI coders. Thus the retention-entropy definition will have the same expectation structure as $\boldsymbol{H}$ [3]. In addition, $\boldsymbol{N}$ will be defined in terms of the microstate uncertainties in physics [6]. Also, a retention constant k_R in cubic meters per retained mass and/or energy will be used for $\boldsymbol{N}$. This retention constant will have a value extracted from UNBH conditions since a UNBH retention-coder provides the maximum expected retention-time per lifespace for the given mass and/or energy. Thus the retention-entropy is defined as follows

$$\boldsymbol{N} = \sum_{i=1}^{V} P[w_i] I_R[w_i] \quad \text{in } m^3 \text{ per } \mathbf{W} \in \{w_1, \ldots, w_\Omega\} \tag{3.5}$$

where $\mathbf{W}$ is a knowledge discrete random variable (or random microstate) with Ω outcomes (or microstate realizations) $\{w_i\}$ and $I_R[w_i]$ is the *retention-information* provided by the outcome w_i and given by the expression

$$I_R(w_i) = k_R \frac{T_R^i}{A_R^i} \log_2(1/P[w_i]) \text{ in } m^3 \text{ per } w_i \tag{3.6}$$

with $P[w_i]$ being the probability of the microstate w_i whose value is driven by the passing of time, T_R^i is the expected retention-time of w_i, A_R^i is the surface area of the volume wherein w_i is retained, and k_R is the retention constant

$$k_R = \frac{\hbar_M^2 G_M^2}{120 c_M^5 \ln 2} = 2.4730 \text{ x } 10^{-133} \quad \text{in } m^5/\text{sec}. \tag{3.7}$$

In addition, for the special case where all the microstates of $\mathbf{W}$ are equally likely and their expected retention-times and retention surfaces are equal it follows that

$$\boldsymbol{N} = k_R \frac{T_R}{A_R} \log_2 \Omega \text{ in } m^3 \text{ per } \mathbf{W} \tag{3.8}$$

The expression for k_R (3.7) is derived using UNBH conditions and under the assumption that all microstates are equally likely. More specifically it is first assumed that the retention-entropy $\boldsymbol{N}$ is equal to the volume or retention-space of a UNBH, thus

$$\boldsymbol{N} = k_R \frac{T_R}{A_R} \log_2 \Omega = V_R \tag{3.9}$$

where V_R is the UNBH volume. Next using the microstate thermodynamic entropy $S=k_M \ln\Omega= k_M \log_2\Omega/\ln 2$ expression in (3.9) where k_M is Boltzmann's constant it is found that

$$\boldsymbol{N} = k_R \frac{T_R S}{A_R k_M} \ln 2 = V_R \tag{3.10}$$

Then using Hawking's black hole thermodynamic entropy $S = A_R k_M c_M^3 / (4\hbar_M G_M)$ in (3.10) the expression

$$\boldsymbol{N} = k_R T_R \frac{c_M^3 \ln 2}{4\hbar_M G_M} = V_R \tag{3.11}$$

is obtained. Next solving expression (3.11) for k_R it follows that

$$k_R = \frac{V_R}{T_R}\frac{4\hbar_M G_M}{c_M^3 \ln 2} \quad (3.12)$$

From equation (A.12) in Appendix A it is then found that for a UNBH the ratio T_R/V_R is the same as the pace of dark c_R expression (1.1) under the assumption that the mass and/or energy starts its retention at $t_M^i = 0$. Thus $T_R / V_R = c_R = 480c_M^2 / \hbar_M G_M$ is used in (3.12) to yield the desired result (3.7). Furthermore, since the UNBH is spherical in shape we use $T_R/A_R = (T_R/V_R)(r_R/3)$ in (3.8) to obtain the equivalent expression

$$\boldsymbol{N} = k_R c_R \frac{r_R}{3}\log_2 \Omega = k_R^{'} m_M^{\mathrm{BH}} \log_2 \Omega \quad \text{in } m^3 \text{ per } \mathbf{W} \quad (3.13)$$

where $r_R = 2G_M m_M^{BH} / c_M^2$ is the Schwarzschild radius for a UNBH, m_M^{BH} is the UNBH mass, and

$$k_R^{'} = \frac{8\hbar_M G_M^2}{3c_M^5 \ln 2} = 7.5046 \text{ x } 10^{-97} \quad \text{in } m^3/\text{kg} \quad (3.14)$$

In addition, to conform to the holographic principle [7] an alternative definition for the retention-entropy can be given in squared meter units. Thus the following holographic retention-entropy $\boldsymbol{N}^h$ is defined

$$\boldsymbol{N}^h = \sum_{i=1}^{V} P[w_i] I_R^h(w_i) \quad \text{in } m^2 \text{ per } \mathbf{W} \in \{w_1,..., w_\Omega\} \quad (3.15)$$

where **W** is a knowledge discrete random variable (random microstate) with Ω outcomes (or microstate realizations) $\{w_i\}$ and $I_R^h(w_i)$ is the *holographic retention-information* provided by the outcome w_i and given by the expression

$$I_R^h(w_i) = k_R \frac{T_R^i}{V_R^i}\log_2(1/P[w_i]) \quad \text{in } m^2 \text{ per } w_\mathrm{i} \quad (3.16)$$

with all the parameters of (3.16) being the same as those in (3.6) except that the retention surface area A_R^i has been replaced with the retention volume V_R^i. For the special case where all the microstates of **W** are equally likely and their expected retention-times and retention volumes are equal it follows that

$$\boldsymbol{N}^h = k_R \frac{T_R}{V_R}\log_2 \Omega \text{ in } m^2 \text{ per } \mathbf{W} \quad (3.17)$$

It should be noted that when (3.17) is equated to the UNBH surface A_R it is once again found, as expected, that k_R is given by (3.7). Furthermore, if we let $T_R/V_R = c_R$ in (3.17), i.e., for a UNBH condition, it follows that

$$\boldsymbol{N}^h = k_R^h \log_2 \Omega \text{ in } m^2 \text{ per } \mathbf{W} \quad (3.18)$$

where k_R^h is the holographic retention constant given by

$$k_R^h = k_R(\mathrm{T}_R / V_R) = 4\hbar_M G_M / c_M^3 \ln 2 = (1920/\ln 2)/c_M c_R = 1.5112 \text{ x } 10^{-69} \ m^2 \text{ per } \mathbf{W} \quad (3.19)$$

The design of a retention-coder is then approached using $\boldsymbol{N}$ (or $\boldsymbol{N}^h$) as a lower lifespace performance bound for the desired knowledge expected retention-time. A retention-coder will be lossless when

$$\boldsymbol{N} \le R_{RC} \le R_R \quad (3.20)$$

where R_{RC} is the retention-coder rate in m^3 (or m^2) per stored mass and/or energy and R_R is the retention rate of some initial retention system. On the other hand, a lossy retention-coder is one that has a R_{RC} that is not achievable, i.e.,

$$0 \le R_{RC} < \boldsymbol{N} . \quad (3.21)$$

4. THE LAWS OF RETENTION

In this section we will find the space dual of the laws of motion in physics, starting first with Newton's Principia [1] and then moving on to its extensions.

4.1 The Retention-Principia

Refer to Fig. 2 A-D where a mass in motion is displayed. This mass will also be called a motion-mass and is given the notation m_M when contrasting it to its space dual, i.e., a retention-**mater** m_R that will be defined later. In Fig. 2A the true nonzero occupancy-space S_0 of m_M is shown where S_0 is said to be governed by a configuration of space certainty. Furthermore, m_M can vary as a function of time, i.e., $m_M(t_M)$, where the time t_M will also be called a motion-time to contrast it with the expected retention-time T_R of retention problems to be discussed later. An example of $m_M(t_M)$ is a space rocket whose mass changes, due to its fuel consumption, as t_M increases. In Fig. 2B we present an idealization of the occupancy-space S_0^1 of $m_M(t_M^1)$ for the motion-time t_M^1. The idealization consists of representing $m_M(t_M^1)$ by a point (shown in the picture as a rectangle) in three dimensional vector space $\mathbf{s}_M=[s_{M,x},s_{M,y},s_{M,z}]$, i.e., $S_0^1 \rightarrow \mathbf{s}_M$. To facilitate the discussion of the space-time duality in physics it will be assumed in this paper, unless specified otherwise, that all motions are in one dimensional space, thus when describing this case the scalar space s_M variable will be used. In addition, any space location used for motion purposes will also be called motion-space to contrast it with the retention-space S_R (in cubic meters) of retention problems to be discussed later. Fig. 2C is similar to Fig. 2B except that it presents $m_M(t_M^2)$ after its space-dislocation (*SD*) from s_M^1 to s_M^2, i.e., $SD = s_M^2 - s_M^1$, and resulting in the lifetime penalty (*LTP*) $LTP = t_M^2 - t_M^1$. The difference of the rectangle horizontal length of Fig. 2C from that of Fig. 2B indicates that $m_M(t_M)$ has paid a lifetime penalty for its space-dislocation. In Fig. 2D the principia model for the movement of m_M is summarized. In Table 1A a summary is provided of well known motion-principia concepts that relate to the motion-principia model of Fig. 2D. These concepts will be contrasted next with those of the retention-principia model.

Refer to Fig. 2 E-H where a retention-mater m_R is displayed which is the space dual of a motion-mass m_M. m_R is given in *Joules.m³/sec* units and is a function of m_M as will be seen shortly. In Fig. 2E the true space distributed occupancy-time T_0 of m_R is shown where T_0 is governed by a passing of time uncertainty. Furthermore, m_R can vary as a function of retention-space S_R in cubic space units, i.e., $m_R(S_R)$. This is the case, for instance, with a UNBH whose m_R increases as its volume $V_R=S_R$ increases when it receives new motion-mass and/or motion-energy. In Fig. 2F we present an idealization of the occupancy-time T_0^1 of $m_R(S_R^1)$ for the initial retention-space S_R^1. The idealization consists of having the retention-mater $m_R(S_R^1)$ characterized by a single expected retention-time T_R^1 (shown in Fig. 2F as a circle), i.e., $T_0^1 \rightarrow T_R^1$. Fig. 2G is similar to Fig. 2F except that it presents $m_R(S_R^2)$ after the time-dislocation (*TD*) from T_R^1 to T_R^2, i.e., $TD = T_R^2 - T_R^1$, and resulting in a lifespace penalty (*LSP*) $LSP = S_R^2 - S_R^1$. The difference in radius of the circle of Fig. 2G from that of Fig. 2F indicates that $m_R(S_R)$ has paid a lifespace penalty for its time-dislocation. For example, in our running example, the UNBH must pay the penalty of increasing its lifespace S_R (or equivalently its volume V_R since $S_R=V_R$) to increase its retention-time from T_R^1 to T_R^2. Furthermore, this lifespace increase is accompanied by a mass increase as noted from the volume-mass relation for a UNBH derived in Appendix A (A.11). In Fig. 2H the principia model for the retention of m_R is summarized. This retention-principia model is the space dual of the motion-principia model of Fig. 2D. Associated with the retention-principia model the most fundamental retention-principia concepts are then summarized in Table 1B. The motion-principia and retention-principia concepts of Table 1A and 1B are now contrasted: 1) the retention-space S_R in m^3 is the space dual of the motion-time t_M where S_R and t_M assume independent variable roles; 2) the retention-time T_R in *sec* is the space dual of the motion-space s_M where T_R and s_M assume dependent variable roles in terms of S_R and t_M, respectively, e.g., from (A.10) a UNBH's retention-time is given by $T_R = c_R V_R = c_R S_R$ when the m_M^{BH}'s retention begins at the zero time instant, i.e., $t_M^i = 0$; 3) the retention-**tempo** $v_R=dT_R/dS_R$ in *sec/m³* is the space dual of the motion-velocity $v_M=ds_M/dt_M$, e.g., for the UNBH $v_R=dT_R/dS_R= c_R$ since $T_R = c_R S_R$; 4) the retention-**pace** $b_R=|v_R|$ in *sec/m³* is the space dual of the motion-speed b_M and for the UNBH it is the same as the pace of dark c_R; 5) the retention-**escalation** $a_R=dv_R/dS_R$ in *sec/m⁶* is the space dual of the motion-acceleration $a_M=dv_M/dt_M$ where $a_R=0$ for the UNBH; 6) the retention-**mater** m_R in *Joule.m³/sec* is the space dual of the motion-mass m_M and for a UNBH is given by $m_R^{BH} = (c_M^2 / c_R) m_M^{BH} = 1.4704 \text{ x } 10^{-47} m_M^{BH}$ *Joule.m³/sec* (derived next); 7) the retention-energy $p_R = m_R v_R$ in *Joules* is the space dual of the motion-momentum $p_M = m_M v_M$ and for a UNBH p_R^{BH} is the same as the UNBH rest motion-energy $E_M^{BH} = m_M^{BH} c_M^2$, thus $E_M^{BH} = m_M^{BH} c_M^2 = p_R^{BH} = m_R^{BH} v_R^{BH} = m_R^{BH} c_R$ (from this condition the aforementioned relationship $m_R^{BH} = \left(c_M^2 / c_R\right) m_M^{BH}$ follows); 8) the retention-pressure $f_R = dp_R / dS_R$ in *Pascals* is the space dual of the motion-force $f_M = dp_M / dt_M$ and for a UNBH after using (A.11) in $f_R = dp_R / dS_R = dE_M / dV_R = c_M^2 / (dV_R / dm_M)$ we obtain $f_R^{BH} = \left(c_M^{12} / 32\pi c_R^2 G_M^3\right) / \left(m_R^{BH}\right)^2 = 468 / \left(m_R^{BH}\right)^2 = \left(c_M^8 / 32\pi G_M^3\right) / \left(m_M^{BH}\right)^2 = 2.1647 \text{ x } 10^{96} / \left(m_M^{BH}\right)^2$ *Pascals*; 9) the kinetic retention-viscosity $KE_R = p_R^2 / 2m_R$ in viscosity units is the space dual of the kinetic motion-energy $KE_M = p_M^2 / 2m_M$ and for a UNBH is given by $KE_R^{BH} = p_R^2 / 2m_R^{BH} = m_R^{BH} c_R^2 / 2 = E_M^{BH} c_R / 2 = m_M^{BH} c_M^2 c_R / 2$ in *viscosity* units; and 10) the retention-**effort** $W_R = \int_{T_R^1}^{T_R^2} f_R(T_R) dT_R$ in viscosity units for a T_R^1 to T_R^2 time-dislocation is the space dual of the motion-work $W_M = \int_{s_M^1}^{s_M^2} f_M(s_M) ds_M$ and for a UNBH yields $W_R^{BH} = \int_{T_R^1}^{T_R^2} f_R^{BH}(T_R) dT_R = \int_{T_R^1}^{T_R^2} (c_M^2 / dV_R / dm_M^{BH}) dT_R = \int_{T_R^1}^{T_R^2} (c_M^2 c_R / dT_R / dm_M^{BH}) dT_R = \int_{m_R(S_R^1)}^{m_R(S_R^2)} c_R^2 dm_R^{BH} = (m_R^{BH}(S_R^2) - m_R^{BH}(S_R^1)) c_R^2$ $= (m_M^{BH}(t_M^2) - m_M^{BH}(t_M^1)) c_M^2 c_R$ in viscosity units. In addition, we have $W_R^{BH} = \int_{m_R^{BH}(S_R^1)}^{m_R^{BH}(S_R^2)} c_R^2 dm_R^{BH} = E_R^{BH}(S_R^2) - E_R^{BH}(S_R^1)$ $= (E_M^{BH}(t_M^2) - E_M^{BH}(t_M^1)) c_R$ where $E_R^{BH}(S_R) = m_R^{BH}(S_R) c_R^2$ and $E_M^{BH}(t_M) = m_M^{BH}(t_M) c_M^2$. The mater-viscosity equivalence equation $E_R(S_R) = m_R(S_R) c_R^2$ is the space dual of the Einstein's mass-energy equivalence equation $E_M(t_M) = m_M(t_M) c_M^2$. From the previous W_R^{BH} expression the *energy-viscosity duality* equation $E_R(S_R) = E_M(t_M)\, c_R$ also arises. Finally it is noted that our earlier use of a UNBH to illustrate retention ideas required us to use physics motion concepts that went far beyond Newton's Principia, e.g., when $E_M = m_M c_M^2$ was used to derive the *mass-mater duality* relation $m_R = \left(c_M^2 / c_R\right) m_M$.

4.2 The Retention Special Relativity

Refer to Fig. 3 where Einstein's special relativity along with the invariant Minkowski spacetime length is displayed in a space-time duality in physics form. In Fig. 3A-C the motion special relativity is shown while in Fig. 3D-F the retention special relativity is displayed. In Fig. 3A the invariant space length l_M (in motion-space units) of motion-spacetime is given while in Fig. 3D the space dual invariant time length l_R (in retention-time units) of retention-spacetime is displayed. In Fig. 3B the motion Lorentz transformations between the observations of two motion inertial (or constant velocity) frames is depicted while in Fig. 3E the space dual retention transformations between the observations of two retention inertial (or constant tempo) frames is given. Finally, in Fig. 3C the motion Einstein invariant energy-momentum equation is shown where $m_M c_M^2$ with m_M at rest (or zero velocity) is motion inertial frame invariant, while in Fig. 3F the space dual retention invariant viscosity-energy equation is presented where $m_R c_R^2$ with m_R at rest (or zero tempo) is retention frame invariant.

4.3 The Retention-Gravidness and Retention-Exalted Law

Refer to Fig. 4 where Newton's gravitational law is displayed in its space-time duality in physics field form. In Fig. 4A-D the motion-gravitational law case is shown and in Fig. 4E-H its space dual is displayed. This space dual is named the retention-**gravidness** law: in Table 2 space-time duality in physics terms that do not already appear in Table 1 are given for ease of reference. In Fig. 4A-B two different motion-masses are shown that *share the same motion-time t_M.* This is reflected in the space-dislocation model of Fig. 4C where both masses are described using the same rectangle length. Furthermore, in Fig. 4C massless but energetic gravitons speeding at the speed of light are displayed that carry the gravitational field in both directions. It is assumed here that the two masses exist in a vacuum. Thus, a lifetime penalty of $LTP = \left|s_M^2 - s_M^1\right| / c_M$ governs the graviton movement. Since this lifetime penalty is the smallest possible one it is the same as the motion-ectropy $\boldsymbol{A}$, i.e., $\boldsymbol{A} = LTP = \left|s_M^2 - s_M^1\right| / c_M$. In Fig. 4D expressions for the gravitational force $f_M^{G1\leftarrow 2}$ acting on the mass m_M^2 and due to the gravitational field G_M^1 of m_M^1 is given. Next in Fig. 4E-H the retention dual is displayed where Fig. 4E-F display two different maters that *share the same retention-space S_R*. For instance, for our UNBH running example m_R^1 can have the center of mass of its associated m_M^1 in the middle of the UNBH while m_R^2 can have the center of mass of its associated m_M^1 just inside the event horizon. Since these two maters share the same retention-space S_R they are shown in the time-dislocation model of Fig. 4G with the same circle radius. Furthermore, in Fig. 4G materless (i.e., m_R=0) but viscositic retention-**gravids** pacing at the pace of dark are displayed carrying the retention-gravidness field (the space dual of the gravitational field) in both directions. It is assumed here that the two maters exist in a black hole (the space dual of a vacuum). Thus, a lower bound lifespace penalty of $\Delta \mathbf{V} = LSP = \left| T_R^2 - T_R^1 \right| / c_R$ governs the gravid retention where $\Delta \mathbf{V}$ denotes a retention-entropy change. Finally in Fig. 4H expressions for the gravidness pressure $f_R^{G1\leftarrow 2}$ acting on m_R^2 and due to the gravidness field $\mathbf{G}_R^1$, in *Viscosity*/(*Joule.meter*^{3}) units, of m_R^1 are given. These space dual expressions surface naturally from UNBH conditions where a special note is made of the fact that the denominator of the gravidness field G_R^1 is inversely proportional to the absolute value of the time-dislocation between the two maters, i.e., $TD = T_R^2 - T_R^1$, taken to a power of 4/3 rather than the power of two for space-dislocation as occurs in the motion case. The derivation of this result begins with the assumption that there are two point 'motion' masses in the UNBH. One is the motion-mass m_M^1 which is modeled as a point mass residing in the center of the UNBH and the other motion-mass m_M^2 residing just inside the event horizon. The motion-gravitation force expressions of Fig. 4D are then used to yield $f_M^{G1\leftarrow 2} = \overline{G}_M m_M^1 m_M^2 / r_R^2$ where r_R is the retention radius of the UNBH. Then dividing this expression by the surface area A_R of the UNBH and using the mass-mater duality expression $m_R^{BH} = \left(c_M^2 / c_R \right) m_M^{BH}$ and $c_R=T_R/V_R$ for a UNBH the expressions of Fig. 4H surface.

Refer to Fig. 5 where Coulomb's electrical law is displayed in its space-time duality in physics field form. In Fig. 5A-D the motion-electrical law case is shown and in Fig. 5E-H its space dual, i.e., the retention-**exalted** law, is displayed. In Fig. 5C massless but energetic photons speeding at the speed of light thru a vacuum are displayed that carry the electric field in both directions. In Fig. 5E-F the space dual of two motion-charges, i.e., the retention-**clogs** q_R^1 and q_R^2, are shown that share the same retention-space S_R. In Fig. 5G the corresponding time-dislocation model is shown with materless but viscositic retention-**portages** (the space dual of a motion-photon) pacing at the pace of dark in a black hole are displayed that carry the retention-**exalted field** (the space dual of the motion-electrical field) in both directions. In Fig. 5H the retention-exalted law is shown and is derived using a similar approach as that suggested earlier to derive the retention-gravidness law of Fig. 4H. Furthermore, when deriving this law the charge-clog duality relationship $q_R = \left(c_M^2 / c_R \right) q_M$ must be used which is found using black hole conditions, as done earlier to derive the mass-mater duality equation $m_R = \left(c_M^2 / c_R \right) m_M$. Finally it is noted that similarly to the $\mathbf{G}_R$ of Fig. 4H the retention-exalted field $\mathbf{E}_R$, in *Viscosity.sec*^{2}/(*C.meter*^{5}) units, of Fig. 5H is inversely proportional to the absolute value of the retention time-dislocation $TD = T_R^2 - T_R^1$ raised to a 4/3 power.

4.4 The Retention Weave-Pellet Duality

Refer to Fig. 6 where the motion wave-particle duality and its space dual are shown. In Fig. 6A the motion-frequency f_M is displayed in motion-wave $\mathbf{Z}_M$ field cycles per second while in Fig. 6C the space dual retention-**fix** f_R is shown in retention-**weave** (the space dual of a motion-wave) $\mathbf{Z}_R$ field cycles per cubic meter. Examples of $\mathbf{Z}_M$ are the motion-electric $\mathbf{E}_M$ and motion-magnetic $\mathbf{B}_M$ fields of a motion-electromagnetic motion-wave while examples of $\mathbf{Z}_R$ are the retention-exalted $\mathbf{E}_R$ and retention-mesmeric $\mathbf{B}_R$ fields of a retention-**exaltmesmeric** retention-weave to be defined shortly. Also in Fig. 6A the motion-wavelength λ_M is given in space-dislocation per motion field cycle while in Fig. 6C the retention-**weavelength** λ_R is given in time-dislocation per retention field cycle. In addition, in Fig. 6A the relation $c_M = f_M \lambda_M$ for massless motion-photons and motion-gravitons is shown while in Fig. 6C the space dual relation $c_R = f_R \lambda_R$ for materless retention-portages and retention-gravids is given. Also, in Fig. 6A the motion wave-particle duality expressions are shown relating p_M and E_M to λ_M and f_M, respectively, via the motion Plank's constant h_M. while in Fig. 6C the retention weave-pellet duality expressions are shown relating p_R and E_R to λ_R and f_R, respectively, via the space dual of Plank's constant h_R. Furthermore, in Fig. 6C motion-retention duality relationships between the p, E, λ and f are advanced where the one half quantum motion-retention duality $h_R = h_M/2$ is highlighted. This quantum duality relationship was found via the Margulus-Leviton theorem [6] expression $T_R E_M = \pi \hbar_M / 2$ where E_M is the minimum average energy needed for a particle's spin up or down state to remain no longer than the retention-time T_R. Thus since λ_R is the same as the time-dislocation per retention field cycle, i.e., a spin up followed by a spin down for this case, it then follows that $\lambda_R=2T_R$, which when substituted in $T_R E_M = \pi\hbar_M / 2$ yields the desired result $\lambda_R E_M = \pi\hbar_M = h_M / 2 = \lambda_R p_R = h_R$. In Fig. 6B the De Broglie wave-particle relations for particles with or without mass is shown while in Fig. 6D the retention weave-pellet relation for pellets with or without mater is given. Finally the frequency-fix duality relation $f_R = (h_M / h_R) f_M c_R$ of Fig. 6C has an interesting interpretation. It is that it relates the Hawking frequency $f_M^H = c_M^3 / 16\pi G_M m_M^{BH}$ of the radiation energy (positive and negative) of a virtual particle pair, where one particle moves into and one moves out of the black hole at the event horizon, to the fix $f_R^F = (h_M / h_R) f_M^H c_R$ of the retention-**ramification** (the space dual of motion-radiation) viscosity (positive and negative) of the space dual of a virtual particle pair, i.e., a **virtual pellet pair**, where one pellet is retained into and the other is retained out of the black hole at the event horizon. In future publications more will be said about the interactions between virtual particle and virtual pellet pairs.

4.5 The Retention Quantum Mechanics and the Retention Uncertainty Principle

Refer to Fig. 7 where Schrodinger's quantum mechanics and Heisenberg's uncertainty principle relations are displayed in their space-time duality in physics form. In Fig. 7A KE_M, PE_M and E_M denote the motion kinetic energy, potential energy and total energy of a particle while in Fig. 7C KE_R, PE_R and E_R denote the kinetic viscosity, potential viscosity and total viscosity of a pellet. Also while PE_M is a function of the motion-space location of a particle, PE_R is a function of the expected retention-time of a pellet. Furthermore, while the squared magnitude of the motion wave function ψ_M, i.e., $|\psi_M|^2$, inform us about the probability of finding a particle at some s_M, the squared magnitude of the retention weave function ψ_R, i.e., $|\psi_R|^2$, inform us about the probability of finding a pellet with some expected retention-time T_R. On the other hand, the space-momentum uncertainty principle expression $\Delta s_M \Delta p_M \geq \hbar_M / 2$ and the 'motion' time-energy uncertainty principle $\Delta t_M \Delta E_M \geq \hbar_M$ of Fig. 7B have as a space dual the 'retention' time-energy uncertainty principle expression $\Delta T_R \Delta p_R \geq \hbar_R / 2$ and the space-viscosity uncertainty principle expression $\Delta S_R \Delta E_R \geq \hbar_R$ of Fig. 7D, respectively. Finally, in Fig. 8A the relativistic motion quantum mechanics expression for a free (PE_M=0) mass is shown while in Fig. 8B the space dual relativistic retention quantum mechanics equations for a free (PE_R=0) mater is presented. Finally, it is noted that the duality expressions of Figs. 7-8 are connected via $h_R=h_M/2$.

4.6 The Retention-Hefty and the Retention-Mesmeric Laws

In Fig. 9 the motion-heaviside and its space dual, i.e., the retention-**hefty,** laws are given. On the other hand, in Fig. 10 the motion magnetic and its space dual, i.e., the retention-**mesmeric**, laws are depicted. From these two figures it is noted that these laws are associated with motion rotations and retention rotations of masses, maters, charges and clogs. The derivation and discussion of these results will be given in a later publication due to space limitations of the current manuscript. It is noted, however, that the **hefty field** $\mathbf{H}_R$ and the **mesmeric field** $\mathbf{B}_R$ are also, as expected, inversely proportional to the absolute value of $TD = T_R^2 - T_R^1$ raised to a 4/3 power.

4.7 The Retention-Gravidhefty and the Retention-Exaltmesmeric Equations

In Fig. 11A-B the motion-gravitoheaviside equations and the motion-electromagnetic equations (Maxwell's equations) are shown. In Fig. 11C-D the retention-**gravidhefty** equations and the retention-**exaltmesmeric** equations are depicted. The derivation and discussion of these results will be given in a later publication due to space limitations of the current manuscript.

4.8 The Retention-Gravidhefty and the Retention-Exaltmesmeric Weaves

In Fig. 12A-B the motion-gravitoheaviside wave equations and the motion-electromagnetic wave equations are shown. In Fig. 12C-D the retention-**gravidhefty** weave equations and the retention-**exaltmesmeric** weave are displayed. The derivation and discussion of these results will be given in a later publication due to space limitations of the current manuscript.

4.9 The Motion General Relativity and the Retention General Relativity Equations

The general relativity case will be addressed in a later publication.

4. CONCLUSIONS

The discovery of the space dual of the laws of motion in physics has been reported in this paper and named the laws of retention. In the laws of motion domain knowledge motion is modulated by electromagnetic and gravitoheaviside fields carried by massless photons and gravitons, respectively, while in the novel laws of retention domain knowledge retention is modulated by exaltmesmeric and gravidhefty fields carried by materless portages and gravids, respectively. The new laws of retention in physics should bring to a recognition system's study, design and implementation a level of sophistication that rivals that presently applied to a communication system's study, design and implementation. In addition, the integrated use of motion and retention laws in latency-information theory (LIT) problems should offer the possibility of deriving synergistic solutions to complex theoretical as well as practical problems. Two novel performance bounds were also introduced. One was motion-ectropy, which advances a lower bound for the lifetime penalty suffered by knowledge due to a motion-space location change or space-dislocation, while the other was retention-entropy, which advances a lower bound for the lifespace penalty suffered by knowledge due to a retention-time interval change or time-dislocation. Novel concepts of particular interest that have surfaced are those of fix and weavelength which are the retention duals of frequency and wavelength, respectively, in motion. These new concepts are of primary interest since they are expected to play a role in recognition systems that emulate that of frequency and wavelength in the study, design, and implementation of modern communication systems. Clearly since the laws of retention is one of the two pillars of the newly discovered space-time duality in physics they promise to have general applicability and also to propel us towards a better understanding of our physical world. Such duality may conceivably address in a rather straight forward manner relevant theoretical questions in physics such as the development of a satisfactory quantum gravity theory as well as the advancement of more reliable predictions about future technology. It is also hoped that in the more general context of LIT, from which it inherently surfaced, the newly discovered space-time duality in physics can serve as a valuable pedagogical tool for superior investigations and guidance of existing and/or to be designed and implemented complex systems.

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APPENDIX A

On the Derivation of the Pace of Dark from UNBH Conditions

The derivation begins with a black hole's power radiation expression

$$P_M^{BH}(t_M) = -\frac{dE_M^{BH}(t_M)}{dt_M} = -c_M^2 \frac{dm_M^{BH}(t_M)}{dt_M} \tag{A.1}$$

where $P_M^{BH}(t_M)$ is the power radiation (the subscript of *M* for 'motion' will be used with physical variables that are normally used in the laws of motion in physics definitions [1], [6] while the subscript *R* for 'retention' will be used with physical variables that are defined for the laws of retention in physics of this paper) and $E_M^{BH}(t_M)$ and $m_M^{BH}(t_M)$ are the energy and mass of a UNBH [6] at the instant of time t_M. $P_M^{BH}(t_M)$ is then noted to be equal to a 'black body' luminance $L_M^{BH}(t_M)$ resulting in the expression

$$P_M^{BH}(t_M) = L_M^{BH}(t_M) = (\pi^2 k_M^4 / 60\hbar_M^3 c_M^2) A_R \left(\Gamma_M^{BH}(t_M)\right)^4 \tag{A.2}$$

where k_M is Boltzmann's constant, $\hbar_M$ is Plank's reduced constant, c_M is the speed of light and $\Gamma_M^{BH}(t_M)$ is the temperature of a UNBH: the radiation frequency of the black body, or equivalently Hawking's radiation frequency $f_M^H(t_M)$ for a black hole, is related to $\Gamma_M^{BH}(t_M)$ via the expression $f_M^H(t_M) = k_M \Gamma_M^{BH}(t_M)/2\hbar_M$. In addition, A_R is the surface of the UNBH retention sphere. Next it is noted that $\Gamma_M^{BH}(t_M)$ is given by the reciprocal of the rate of change of the UNBH thermodynamic entropy $S_M^{BH}(t_M)$ with respect to $E_M^{BH}(t_M)$ where the $S_M^{BH}(t_M)$ is given by the Hawking entropy [6]. Thus

$$\Gamma_M^{BH}(t_M) = \left(\partial S_M^{BH}(t_M) / \partial E_M^{BH}(t_M)\right)^{-1} \tag{A.3}$$

$$S_M^{BH}(t_M) = \frac{k_M c_M^3}{4\hbar_M G_M} A_R : \text{Hawking's entropy} \tag{A.4}$$

Next using Schwarzschild's radius in the expression for A_R in (A.4) and then replacing $m_M^{BH}(t_M)$ with its energy equivalence one obtains the following expression for $S_M^{BH}(t_M)$ as a function of $E_M^{BH}(t_M)$:

$$S_M^{BH}(t_M) = \frac{k_M c_M^3}{4\hbar_M G_M} A_R = \frac{k_M \pi c_M^3}{\hbar_M G_M}\left(\frac{2G_M m_M^{BH}(t_M)}{c_M^2}\right)^2 = \frac{k_M 4\pi G_M}{\hbar_M c_M^5}\left(E_M^{BH}(t_M)\right)^2 \tag{A.5}$$

Next using (A.5) in the evaluation of (A.3) one finds

$$\Gamma_M^{BH}(t_M) = \frac{\hbar_M c_M^3}{8\pi k_M G_M m_M^{BH}(t_M)} \tag{A.6}$$

Using (A.6) in (A.2) and equating the result with (A.1) the following nonlinear differential equation is derived

$$\frac{dm_M^{BH}(t_M)}{dt_M} + \frac{\hbar_M c_M^4}{15360\pi G_M^2 \left(m_M^{BH}(t_M)\right)^2} = 0 \tag{A.7}$$

The solution to this differential equation then yields the following analytical result

$$\left(m_M^{BH}(t_M)\right)^3 = \left(m_M^{BH}(t_M^i)\right)^3 - (\hbar_M c_M^4 / 5120\pi G_M^2) t_M \tag{A.8}$$

where $m_M^{BH}(t_M^i)$ is the initial UNBH mass. One then sets expression (A.8) to zero to find the final time t_M^f when the black hole ends its existence, i.e.,

$$t_M^f = \frac{5120\pi G_M^2}{\hbar_M c_M^4}\left(m_M^{BH}(t_M^i)\right)^3 = \frac{5120\pi G_M^2}{\hbar_M c_M^4}\left(\frac{c_M^2 r_R}{2G_M}\right)^3 = \frac{640\pi c_M^2}{\hbar_M G_M}(r_R)^3 = \frac{480 c_M^2}{\hbar_M G_M} V_R \tag{A.9}$$

where r_R and V_R are the retention radius and volume, respectively, of the UNBH at the initial time of t_M^i. The expected retention-time T_R for the UNBH is then given by the expression

$$T_R = t_M^f - t_M^i = (480 c_M^2 / \hbar_M G_M) V_R - t_M^i \tag{A.10}$$

$$V_R = 4\pi (r_R)^3 / 3 = 4\pi \left(2G_M m_M^{BH}(t_M^i) / c_M^2\right)^3 / 3 = 4\pi \left(2G_M E_M^{BH}(t_M^i) / c_M^4\right)^3 / 3 \tag{A.11}$$

The rate of change of expected retention-time T_R with respect to the retention-space V_R is then derived from (A.10) to give us the sought after pace of dark c_R for a UNBH, i.e.,

$$c_R = dT_R / dV_R = 480 c_M^2 / \hbar_M G_M = (T_R + t_M^i) / V_R. \tag{A.12}$$

Table 1. A) Newton's Principia; B) The Retention-Principia

A	B
t_M: motion-time (or lifetime) in *sec* units	S_R: retention-space (or lifespace) in *meter*^{3} units
s_M: motion-space in *meter* units	T_R: retention-time in *sec* units
v_M: motion-velocity in *meters/sec* units	v_R: retention-**tempo** in *sec/meter*^{3} units
b_M: motion-speed in *meters/sec* units	b_R: retention-**pace** in *sec/meter*^{3} units
a_M: motion-acceleration in *meters/sec*^{2} units	a_R: retention-**escalation** in *sec/meter*^{6} units
m_M: motion-mass in *kg* units	m_R: retention-**mater** in kg_R=*Joule.meter*^{3}*/sec* units
p_M: motion-momentum in *Newtons.secs* units	p_R: retention-energy in *Joule* units
f_M: motion-force in *Newtons* units	f_R: retention-pressure in *Pascal* units
KE_M: kinetic motion-energy in *Joule* units	KE_R: kinetic retention-viscosity in *Viscosity* units
W_M: motion-work in *Joule* units	W_R: retention-**effort** in *Viscosity* units
$v_M = ds_M / dt_M$	$v_R = dT_R / dS_R$
$b_M = \|v_M\|$	$b_R = \|v_R\|$
$a_M = dv_M / dt_M$	$a_R = dv_R / dS_R$
$p_M = m_M v_M$	$p_R = m_R v_R$
$f_M = dp_M / dt_M$	$f_R = dp_R / dS_R$
$KE_M = p_M^2 / 2m_M$	$KE_R = p_R^2 / 2m_R$
$W_M = \int_{s_M^1}^{s_M^2} f_M(s_M) ds_M$	$W_R = \int_{T_R^1}^{T_R^2} f_R(T_R) dT_R$

Table 2. A) Additional Motion Terminology; B) Retention Space Dual Terminology

A	B
Motion-Gravitational	Retention-Gravidness
Motion-Electrical	Retention-Exalted
Motion-Heaviside	Retention-Hefty
Motion-Magnetic	Retention-Mesmeric
Motion-Frequency	Retention-Fix
Motion-Wave	Retention-Weave
Motion-Wavelength	Retention-Weavelength
Motion-Particle	Retention-Pellet
Motion-Radiation	Retention-Ramification
Motion-Photon	Retention-Portage
Motion-Graviton	Retention-Gravid
Motion-Charge	Retention-Clog

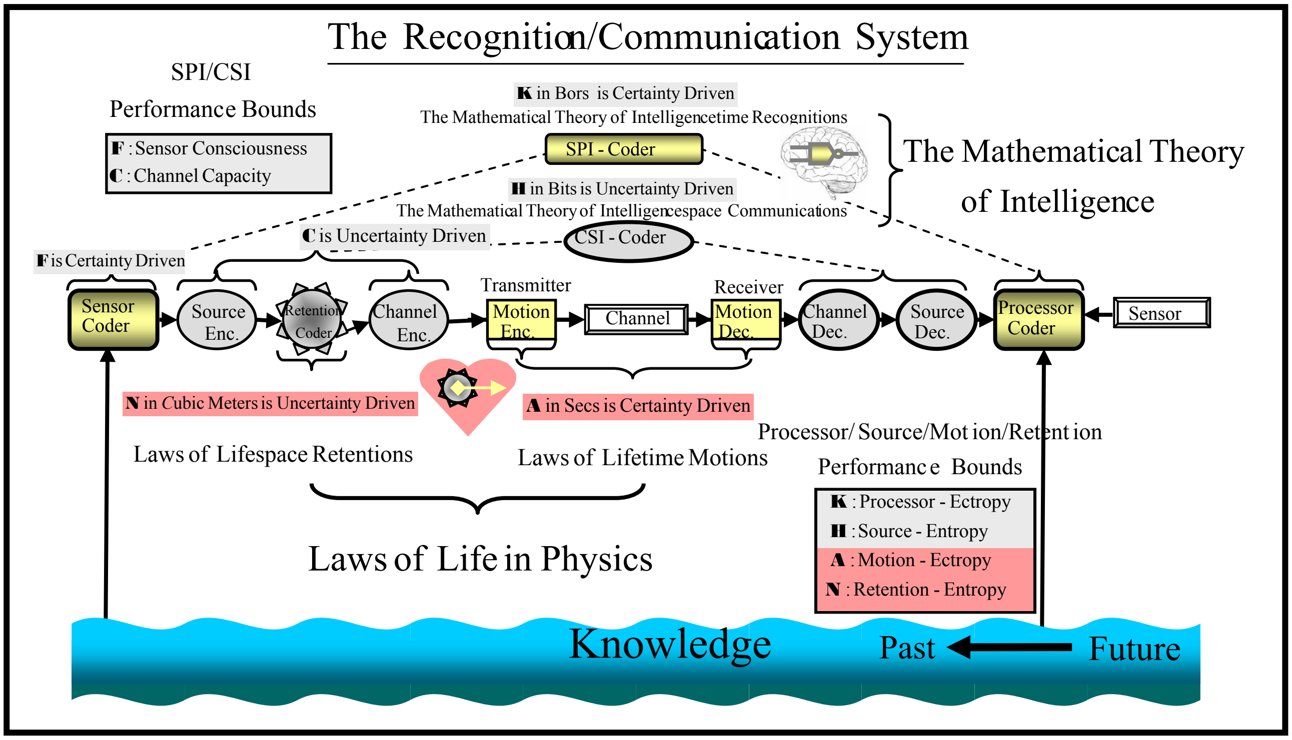


Fig. 1 The Recognition/Communication System in Latency-Information Theory

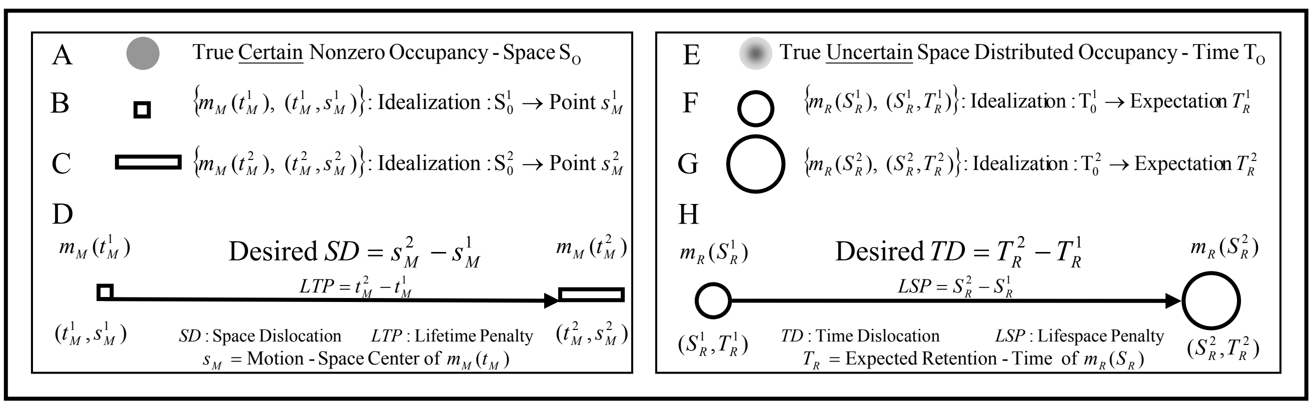


Fig. 2 A-D) The Motion-Principia Model; E-H) The Retention-Principia Model

A

$$l_M^2 = (s_M^2 - s_M^1)^2 - \left(c_M(t_M^2 - t_M^1)\right)^2 = (\Delta s_M)^2 - \left(c_M \Delta t_M\right)^2$$

B

$$\begin{bmatrix} \Delta s_M \\ \Delta t_M \end{bmatrix} = \gamma_M \begin{bmatrix} 1 & \Delta b_M \\ \Delta b_M / c_M^2 & 1 \end{bmatrix} \begin{bmatrix} \Delta s_M' \\ \Delta t_M' \end{bmatrix} \qquad \begin{bmatrix} \Delta s_M' \\ \Delta t_M' \end{bmatrix} = \gamma_M \begin{bmatrix} 1 & -\Delta b_M \\ -\Delta b_M / c_M^2 & 1 \end{bmatrix} \begin{bmatrix} \Delta s_M \\ \Delta t_M \end{bmatrix}$$

$$\gamma_M = 1/\sqrt{1 - \Delta b_M^2 / c_M^2} \geq 1$$

Δb_M : Speed Difference Between the two Motion Inertial (constant velocity) Frames

C

$$(m_M c_M^2)^2 = (E_M)^2 - (p_M c_M)^2$$

m_M = Rest (zero velocity) motion - mass

$$E_M = \gamma_M m_M c_M^2 \qquad p_M = \gamma_M m_M v_M$$

D

$$l_R^2 = (T_R^2 - T_R^1)^2 - \left(c_R(S_R^2 - S_R^1)\right)^2 = (\Delta T_R)^2 - \left(c_R \Delta S_R\right)^2$$

E

$$\begin{bmatrix} \Delta T_R \\ \Delta S_R \end{bmatrix} = \gamma_R \begin{bmatrix} 1 & \Delta b_R \\ \Delta b_R / c_R^2 & 1 \end{bmatrix} \begin{bmatrix} \Delta T_R' \\ \Delta S_R' \end{bmatrix} \qquad \begin{bmatrix} \Delta T_R' \\ \Delta S_R' \end{bmatrix} = \gamma_R \begin{bmatrix} 1 & -\Delta b_R \\ -\Delta b_R / c_R^2 & 1 \end{bmatrix} \begin{bmatrix} \Delta T_R \\ \Delta S_R \end{bmatrix}$$

$$\gamma_R = 1/\sqrt{1 - \Delta b_R^2 / c_R^2} \geq 1$$

Δb_R : Pace Difference Between th e two Retention Inertial (constant tempo) Frames

F

$$(m_R c_R^2)^2 = (E_R)^2 - (p_R c_R)^2$$

m_R = Rest (zero tempo) retention - mater

$$E_R = \gamma_R m_R c_R^2 \qquad p_R = \gamma_R m_R v_R$$

Fig. 3 A-C) Motion Special Relativity; D-F) Retention Special Relativity

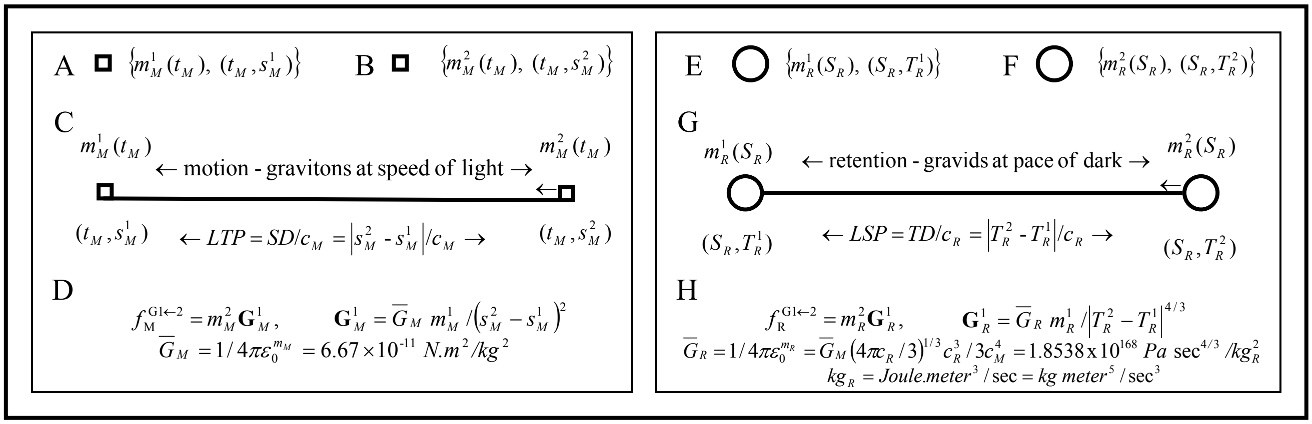


Fig. 4 A-D) The Motion-Gravitational Law; E-H) The Retention-Gravidness Law

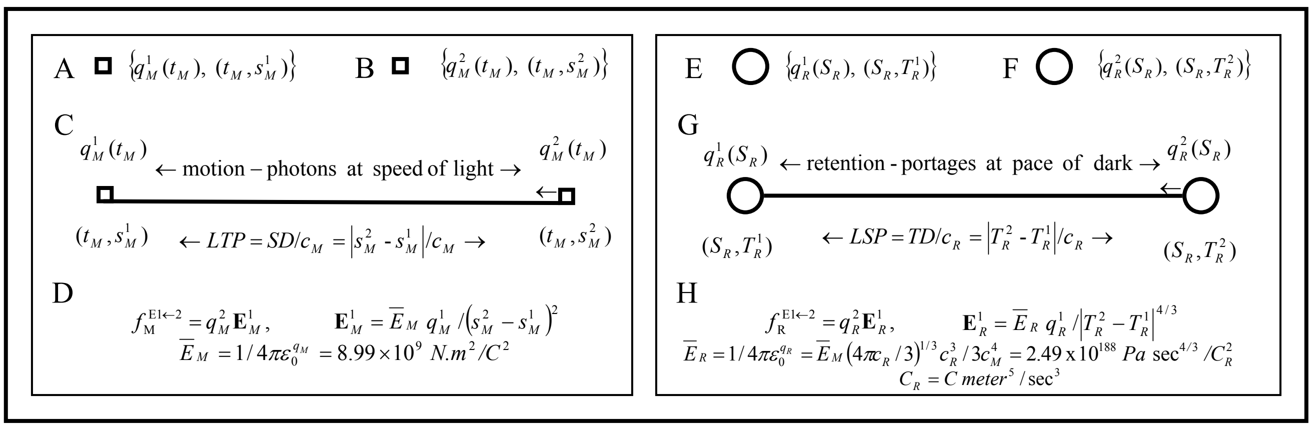


Fig. 5 A-D) The Motion-Electrical Law; E-H) The Retention-Exalted Law

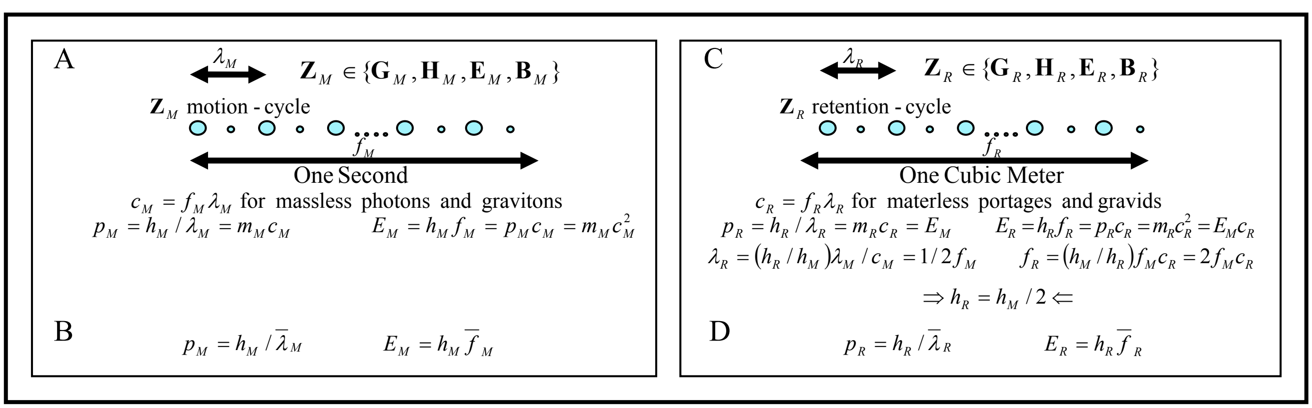


Fig. 6 A-B) The Motion Wave-Particle Duality; C-D) The Retention Weave-Pellet Duality;

A

$$KE_M + PE_M = E_M$$

$$KE_M = p_M^2 / 2m_M = (h_M / \overline{\lambda}_M)^2 / 2m_M, \quad E_M = h_M \overline{f}_M,$$

$$-(\hbar_M^2 / 2m_M) d^2\psi_M / d(s_M)^2 + PE_M \psi_M = E_M \psi_M$$

$$\Psi_M(s_M, t_M) = \psi(s_M) e^{-j2\pi \overline{f}_M t_M}$$

B

$$\Delta s_M \Delta p_M \geq \hbar_M / 2 \qquad \Delta t_M \Delta E_M \geq \hbar_M$$

C

$$KE_R + PE_R = E_R$$

$$KE_R = p_R^2 / 2m_R = (h_R / \overline{\lambda}_R)^2 / 2m_R, \quad E_R = h_R \overline{f}_R,$$

$$-(\hbar_R^2 / 2m_R) d^2\psi_R / d(T_R)^2 + PE_R \psi_R = E_R \psi_R$$

$$\Psi_R(T_R, S_R) = \psi(T_R) e^{-j2\pi \overline{f}_R S_R}$$

D

$$\Delta T_R \Delta p_R \geq \hbar_R / 2 \qquad \Delta S_R \Delta E_R \geq \hbar_R$$

Fig. 7 A) Motion Quantum Mechanics; B) Motion Uncertainty Principle;
C) Retention Quantum Mechanics; D) Retention Uncertainty Principle

A

For Free ($PE_M = 0$) Mass

$$(p_M c_M)^2 + (m_M c_M^2)^2 = (E_M)^2$$

$$p_M^2 / 2m_M = (\hbar_M^2 / 2m_M)(2\pi / \overline{\lambda}_M)^2 \qquad E_M^2 / 2m_M c_M^2 = (\hbar_M^2 / 2m_M c_M^2)(2\pi \overline{f}_M)^2$$

$$(2\pi / \overline{\lambda}_M)^2 \boldsymbol{\psi}_M = -\partial^2 \boldsymbol{\psi}_M / \partial(s_M)^2 \qquad (2\pi \overline{f}_M)^2 \boldsymbol{\psi}_M = -\partial^2 \boldsymbol{\psi}_M / \partial(t_M)^2$$

$$(\hbar_M^2 / 2m_M)\partial^2 \boldsymbol{\psi}_M / \partial(s_M)^2 - (m_M c_M^2 / 2)\boldsymbol{\psi}_M = (\hbar_M^2 / 2m_M c_M^2)\partial^2 \boldsymbol{\psi}_M / \partial(t_M)^2$$

B

For Free ($PE_R = 0$) Mater

$$(p_R c_R)^2 + (m_R c_R^2)^2 = (E_R)^2$$

$$p_R^2 / 2m_R = (\hbar_R^2 / 2m_R)(2\pi / \overline{\lambda}_R)^2 \qquad E_R^2 / 2m_R c_R^2 = (\hbar_R^2 / 2m_R c_R^2)(2\pi \overline{f}_R)^2$$

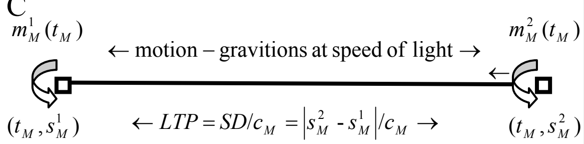
$$(2\pi / \overline{\lambda}_R)^2 \boldsymbol{\psi}_R = -\partial^2 \boldsymbol{\psi}_R / \partial(T_R)^2 \qquad (2\pi \overline{f}_R)^2 \boldsymbol{\psi}_R = -\partial^2 \boldsymbol{\psi}_R / \partial(S_R)^2$$

$$(\hbar_R^2 / 2m_R)\partial^2 \boldsymbol{\psi}_R / \partial(T_R)^2 - (m_R c_R^2 / 2)\boldsymbol{\psi}_R = (\hbar_R^2 / 2m_R c_R^2)\partial^2 \boldsymbol{\psi}_R / \partial(S_R)^2$$

Fig. 8 A) Motion Relativistic Quantum Mechanics; B) Retention Relativistic Quantum Mechanics

A $\{m_M^1(t_M),\ (t_M, s_M^1)\}$ B $\{m_M^2(t_M),\ (t_M, s_M^2)\}$

C



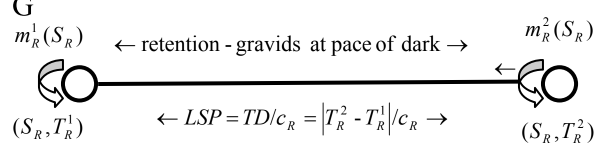
D

$$f_M^{H1\leftarrow 2} = m_M^2 v_M^2 \times \mathbf{H}_M^1, \quad \mathbf{H}_M^1 = \overline{H}_M m_M^1 v_M^1 \times \hat{\mathbf{s}}_M / (s_M^2 - s_M^1)^2$$

$$\overline{H}_M = \overline{G}_M / c_M^2 = \mu_0^{m_M} / 4\pi = 7.46 \times 10^{-28}\ N / (kg / \sec)^2$$

E $\{m_R^1(S_R),\ (S_R, T_R^1)\}$ F $\{m_R^2(S_R),\ (S_R, T_R^2)\}$

G



H

$$f_R^{H1\leftarrow 2} = m_R^2 v_R^2 \times_R \mathbf{H}_R^1, \quad \mathbf{H}_R^1 = \overline{H}_R m_R^1 v_R^1 \times_R \hat{\mathbf{s}}_R / |T_R^2 - T_R^1|^{4/3}$$

$$\overline{H}_R = \overline{G}_R / c_R^2 = \mu_0^{m_R} / 4\pi = 4.9619 \text{x} 10^{40}\ Pa.\sec^{-2/3} / (kg_R / m^3)^2$$

Fig. 9 A-D) The Motion-Heaviside Law; E-H) The Retention-Hefty Law

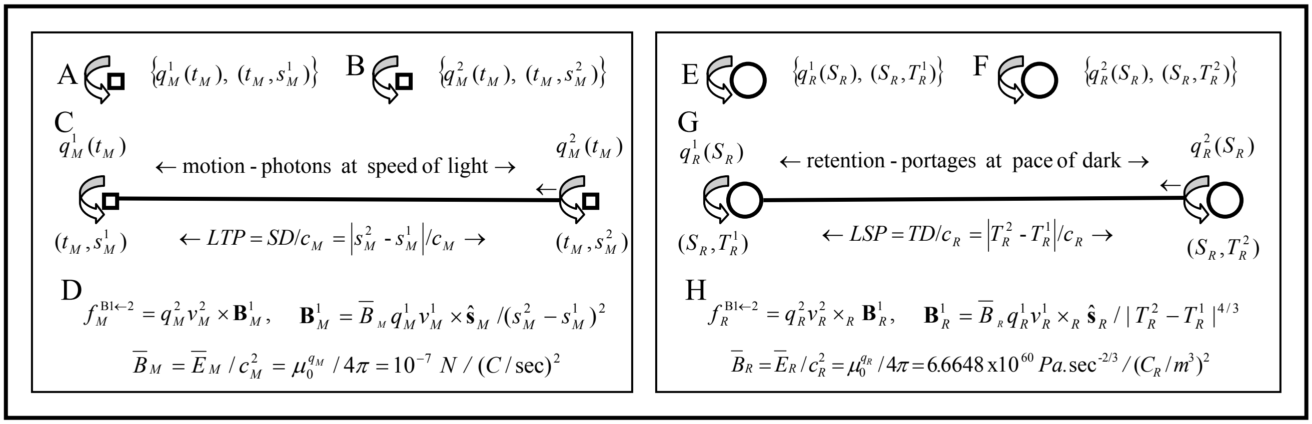


Fig. 10 A-D) The Motion-Magnetic Law; E-H) The Retention-Mesmeric Law

A

$$\nabla . \mathbf{G}_M = \rho^{m_M} / \varepsilon_0^{m_M} \quad \nabla . \mathbf{H}_M = 0 \quad \nabla \times \mathbf{G}_M = -\partial \mathbf{H}_M / \partial t_M$$
$$\nabla \times \mathbf{H}_M = \mu_0^{m_M} \mathbf{J}^{m_M} + \mu_0^{m_M} \varepsilon_0^{m_M} \partial \mathbf{G}_M / \partial t_M$$
$$c_M^2 = 1/\mu_0^{m_M} \varepsilon_0^{m_M}$$

B

$$\nabla . \mathbf{E}_M = \rho^{q_M} / \varepsilon_0^{q_M} \quad \nabla . \mathbf{B}_M = 0 \quad \nabla \times \mathbf{E}_M = -\partial \mathbf{B}_M / \partial t_M$$
$$\nabla \times \mathbf{B}_M = \mu_0^{q_M} \mathbf{J}^{q_M} + \mu_0^{q_M} \varepsilon_0^{q_M} \partial \mathbf{E}_M / \partial t_M$$
$$c_M^2 = 1/\mu_0^{q_M} \varepsilon_0^{q_M}$$

C

$$\nabla ._R \mathbf{G}_R = \rho^{m_R} / \varepsilon_0^{m_R} \quad \nabla ._R \mathbf{H}_R = 0 \quad \nabla \times_R \mathbf{G}_R = -\partial \mathbf{H}_R / \partial S_R$$
$$\nabla \times_R \mathbf{H}_R = \mu_0^{m_R} \mathbf{J}^{m_R} + \mu_0^{m_R} \varepsilon_0^{m_R} \partial \mathbf{G}_R / \partial S_R$$
$$c_R^2 = 1/\mu_0^{m_R} \varepsilon_0^{m_R}$$

D

$$\nabla ._R \mathbf{E}_R = \rho^{q_R} / \varepsilon_0^{q_R} \quad \nabla ._R \mathbf{B}_R = 0 \quad \nabla \times_R \mathbf{E}_R = -\partial \mathbf{B}_R / \partial S_R$$
$$\nabla \times_R \mathbf{B}_R = \mu_0^{q_R} \mathbf{J}^{q_R} + \mu_0^{q_R} \varepsilon_0^{q_R} \partial \mathbf{E}_R / \partial S_R$$
$$c_R^2 = 1/\mu_0^{q_R} \varepsilon_0^{q_R}$$

Fig. 11 A) The Motion-Gravitoheaviside Equations; B) The Motion-Electromagnetic Equations;
C) The Retention -Gravidhefty Equations; D) The Retention-Exaltmesmeric Equations

A

$$c_M^2 \partial^2 G_M / \partial (s_M)^2 = \partial^2 G_M / \partial (t_M)^2 \Rightarrow G_M = G^M \sin(\overline{k}_M\ s_M - \omega_M t_M)$$
$$c_M^2 \partial^2 H_M / \partial (s_M)^2 = \partial^2 H_M / \partial (t_M)^2 \Rightarrow H_M = H^M \sin(\overline{k}_M\ s_M - \omega_M t_M)$$
$$\overline{k}_M = 2\pi / \lambda_M \quad \omega_M = 2\pi f_M \quad c_M = f_M \lambda_M = G^M / H^M$$

B

$$c_M^2 \partial^2 E_M / \partial (s_M)^2 = \partial^2 E_M / \partial (t_M)^2 \Rightarrow E_M = E^M \sin(\overline{k}_M\ s_M - \omega_M t_M)$$
$$c_M^2 \partial^2 B_M / \partial (s_M)^2 = \partial^2 B_M / \partial (t_M)^2 \Rightarrow B_M = B^M \sin(\overline{k}_M\ s_M - \omega_M t_M)$$
$$\overline{k}_M = 2\pi / \lambda_M \quad \omega_M = 2\pi f_M \quad c_M = f_M \lambda_M = E^M / B^M$$

C

$$c_R^2 \partial^2 G_R / \partial (T_R)^2 = \partial^2 G_R / \partial (S_R)^2 \Rightarrow G_R = G^R \sin(\overline{k}_R\ T_R - \omega_R S_R)$$
$$c_R^2 \partial^2 H_R / \partial (T_R)^2 = \partial^2 H_R / \partial (S_R)^2 \Rightarrow H_R = H^R \sin(\overline{k}_R\ T_R - \omega_R S_R)$$
$$\overline{k}_R = 2\pi / \lambda_R \quad \omega_R = 2\pi f_R \quad c_R = f_R \lambda_R = G^R / H^R$$

D

$$c_R^2 \partial^2 E_R / \partial (T_R)^2 = \partial^2 E_R / \partial (S_R)^2 \Rightarrow E_R = E^R \sin(\overline{k}_R\ T_R - \omega_R S_R)$$
$$c_R^2 \partial^2 B_R / \partial (T_R)^2 = \partial^2 B_R / \partial (S_R)^2 \Rightarrow B_R = B^R \sin(\overline{k}_R\ T_R - \omega_R S_R)$$
$$\overline{k}_R = 2\pi / \lambda_R \quad \omega_R = 2\pi f_R \quad c_R = f_R \lambda_R = E^R / B^R$$

Fig. 12 A) The Motion-Gravitoheaviside Waves; B) The Motion-Electromagnetic Waves;
C) The Retention-Gravidhefty Weaves; D) The Retention Exaltmesmeric Weaves